MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XC. SUPPLEMENTARY NUMBER. NO. 9

The Dissociation Formula according to the Fermi-Dirac Statistics.

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1. The equation determining the high-temperature ionisation of a gas, first applied in a particular case in an astrophysical problem by Lindemann * and later widely used by Saha † and others in the theory of stellar spectra, has been the subject of various methods of derivation. The most rigorous of these, according to classical statistical mechanics, is due to R. H. Fowler,‡ and expositions of this and other methods have recently been given by Pannekoek.§ A derivation according to the new statistics of Fermi || and Dirac ¶ has been given by R. H. Fowler,** but his results are scarcely in a form suitable for immediate application. The precise form of the ionisation formula in one particular case has been obtained recently by S. Chandrasekhar †† on the basis of the expression for the entropy of a gas given by Fermi, and has been applied by him to discuss the state of ionisation in dense stars. Chandrasekhar's formula is, however, somewhat complicated and his method does not easily lend itself to other cases.

I have recently noticed that it is possible to obtain the general condition of dissociative equilibrium according to the Fermi-Dirac statistics in a very simple form. The proof depends on first principles only, and involves no special technique. Moreover, the new formulæ show readily when the gas or one or more systems of components in

* *Phil. Mag.*, **38**, 676, 1919.
† *Phil. Mag.*, **40**, 472, 809, 1920; *Proc. Roy. Soc.*, **99A**, 135, 1921.
‡ *Phil. Mag.*, **45**, 1, 1923; *Statistical Mechanics* (1929), chap. v.
§ *Handbuch der Astrophysik*, Bd. iii., 1, Kap. 3, 1930.
|| *Zeit. für Phys.*, **36**, 902, 1926.
¶ *Proc. Roy. Soc.*, **112**, A, 672, 1926.
** *Statistical Mechanics* (1929), chap. xxi. p. 538.
†† *Phil. Mag.*, **9**, 292, 1930.

the gas is degenerate and when it is non-degenerate, in the sense of Sommerfeld; * and they reduce immediately to the well-known " Saha " form when there is no degeneracy. My proof contains nothing new in principle, and will indeed be superfluous to those already acquainted with the subject. But the formulæ are novel in the form I give, and lend themselves to certain applications which I have in view. I therefore think it may be useful to astrophysicists generally if I put them on record.

2. *Enunciation of the Dissociation Formulæ.*—I first state the formulæ without proof.

Consider a gas-mixture in which constituents C_1, C_2, . . . C_r, . . . react reversibly to produce constituents C_1', C_2', . . . $C_{r'}$, . . . according to the chemical equation

$$\sum_r n_r C_r = \sum_{r'} n_{r'} C_{r'}, + \chi \quad . \quad . \quad . \quad (0)$$

the notation indicating that χ is the energy *evolved* when n_r molecules of C_r, etc., react to form $n_{r'}$ molecules of $C_{r'}$, etc. The n_r's and $n_{r'}$'s are integers, and χ is expressed in ergs. Let the gas-mixture when in dissociative equilibrium consist of a volume V, at temperature T, containing N_r molecules of C_r, etc., $N_{r'}$ molecules of $C_{r'}$, etc. Let m_r be the mass of the molecule of C_r, $m_{r'}$ that of the molecule of $C_{r'}$. Let q_r be the statistical weight of the molecule of C_r, and similarly $q_{r'}$ for $C_{r'}$. Let h denote Planck's constant, k Boltzmann's constant.

Then the condition of dissociative equilibrium is obtained by eliminating quantities α_1, . . . α_r, . . . α_1', . . . $\alpha_{r'}$, . . . *from the equations*

$$N_r = \frac{2\pi V q_r}{h^3}(2m_r kT)^{\frac{3}{2}} \int_0^\infty \frac{x^{\frac{1}{2}}dx}{e^{\alpha_r + x} + 1}, \quad (r = 1, 2 \ldots) \quad . \quad (1)$$

$$N_{r'} = \frac{2\pi V q_{r'}}{h^3}(2m_{r'} kT)^{\frac{3}{2}} \int_0^\infty \frac{x^{\frac{1}{2}}dx}{e^{\alpha_{r'} + x} + 1}, \quad (r' = 1, 2 \ldots) \quad (2)$$

$$\sum_r n_r \alpha_r = \sum_{r'} n_{r'} \alpha_{r'} + \frac{\chi}{kT} \quad . \quad . \quad . \quad (3)$$

Since we have exactly as many equations of type (1) as quantities α_r, and exactly as many equations of type (2) as quantities $\alpha_{r'}$, it follows that we have one more equation than the total number of the quantities α_r, $\alpha_{r'}$, so that the elimination is just possible. The eliminant is the condition of dissociative equilibrium.†

If more than one chemical reaction is possible amongst the substances C_r, $C_{r'}$ we add for each additional reaction an equation of type (3) formed on the basis of the corresponding chemical equation, and obtain another eliminant. In the enumeration of the different kinds of molecules the different excited forms of any one molecule must be

* *Zeit. für Phys.*, **47**, 1, 1928.

† For the physical meaning of the α's as chemical potentials, see equation (34), § 6, below.

counted as separate species. These then come under the above scheme. Thus, if a molecule C can exist in the forms $C^{(1)}$, $C^{(2)}$ of energies χ_1 and χ_2, then we introduce quantities $\alpha^{(1)}$, $\alpha^{(2)}$ connected by the equation

$$\alpha^{(1)} = \alpha^{(2)} + \frac{\chi_1 - \chi_2}{kT},$$

and eliminate $\alpha^{(1)}$ and $\alpha^{(2)}$ between this and the integrals for $N^{(1)}$ and $N^{(2)}$.

3. For practical applications we require approximations to the definite integrals for the cases of (*a*) non-degeneracy, (*b*) degeneracy.

(*a*) The system is non-degenerate with respect to C_r when

$$\frac{N_r h^3}{2\pi V(2m_r kT)^{\frac{3}{2}} q_r} << 1 \quad . \quad . \quad . \quad . \quad (4)$$

(Sommerfeld's criterion). In that case α_r is large and positive and we have approximately

$$\int_0^\infty \frac{x^{\frac{1}{2}} dx}{e^{\alpha_r + x} + 1} = \tfrac{1}{2}\pi^{\frac{1}{2}} e^{-\alpha_r} \quad . \quad . \quad . \quad (5)$$

Hence for non-degeneracy

$$N_r = \frac{V q_r}{h^3}(2\pi m_r kT)^{\frac{3}{2}} e^{-\alpha_r} \quad . \quad . \quad . \quad (6)$$

(*b*) The system is degenerate with respect to C_r when

$$\frac{N_r h^3}{2\pi V(2m_r kT)^{\frac{3}{2}} q_r} >> 1 \quad . \quad . \quad . \quad . \quad (4')$$

(Sommerfeld's criterion). In that case α_r is large and negative and we have approximately *

$$\int_0^\infty \frac{x^{\frac{1}{2}} dx}{e^{\alpha_r + x} + 1} = \tfrac{2}{3}(-\alpha_r)^{\frac{3}{2}}\left[1 + \frac{\pi^2}{8\alpha_r{}^2}\right] \quad . \quad . \quad (5')$$

Hence for degeneracy

$$N_r = \frac{2\pi V q_r}{h^3}(2m_r kT)^{\frac{3}{2}} \tfrac{2}{3}(-\alpha_r)^{\frac{3}{2}}\left[1 + \frac{\pi^2}{8\alpha_r{}^2}\right] \quad . \quad . \quad (6')$$

From equations of the type (6) and (6′) we may solve for the α_r's and $\alpha_{r'}$''s, insert in equation (3), and thus obtain explicitly the dissociation equation. Case (*a*) corresponds to small partial densities and high temperatures, case (*b*) to large partial densities and low temperatures. It is not, however, the density or temperature *per se* which determines degeneracy, but rather the ratio of the density N_r/V to $(m_r T)^{\frac{3}{2}}$.

4. *Applications.*—(*a*) As a first example of the formulæ we will derive the well-known "Saha" equation for the ionisation of an atom A capable of existing in the state of lowest energy $A^{(1)}$, its ionised form having a state of lowest energy $A_+{}^{(1)}$. Let χ be the ionisation potential.

* Brillouin, *Statistiques Quantiques* (1930), p. 384.

Then, if $c^{(1)}$, $c_+^{(1)}$ are the atomic concentrations (numbers per cm.^{3}), we have for non-degeneracy by (6)

$$c^{(1)} = \frac{q^{(1)}(2\pi m^{(1)}kT)^{\frac{3}{2}}}{h^3}e^{-\alpha^{(1)}} \quad . \quad . \quad . \quad (7)$$

$$c_+^{(1)} = \frac{q_+^{(1)}(2\pi m_+^{(1)}kT)^{\frac{3}{2}}}{h^3}e^{-\alpha_+^{(1)}} \quad . \quad . \quad . \quad (8)$$

For the concentration of electrons c_e we have

$$c_e = \frac{q_e(2\pi m_e kT)^{\frac{3}{2}}}{h^3}e^{-\alpha_e} \quad . \quad . \quad . \quad (9)$$

where q_e, the statistical weight of the electron, is to be taken as 2, owing to the spin. Since the chemical equation is

$$A_+^{(1)} + e = A^{(1)}, \quad + \chi, \quad . \quad . \quad . \quad (10)$$

we have, by use of (3),

$$\alpha_+^{(1)} + \alpha_e = \alpha^{(1)} + \frac{\chi}{kT} \quad . \quad . \quad . \quad (11)$$

Hence by (7), (8), (9), and (11), on eliminating the α's,

$$\frac{c_+^{(1)}c_e}{c^{(1)}} = \left(\frac{m_+^{(1)}m_e}{m^{(1)}}\right)^{\frac{3}{2}}\frac{(2\pi kT)^{\frac{3}{2}}}{h^3}\frac{q_+^{(1)}q_e}{q^{(1)}}e^{-\chi/kT} \quad . \quad . \quad (12)$$

Since we may put $m_+^{(1)} = m^{(1)}$, approximately, this reduces to the well-known form on expressing the concentrations in terms of partial pressures by the formulæ $p^{(1)} = c^{(1)}kT$, $p_+^{(1)} = c_+^{(1)}kT$ which are valid for non-degeneracy.

To take account of an excited state, say $A^{(2)}$, of energy $\chi^{(2)} - \chi^{(1)}$ above the state $A^{(1)}$, we simply add the equations

$$c^{(2)} = \frac{q^{(2)}(2\pi m^{(2)}kT)^{\frac{3}{2}}}{h^3}e^{+\alpha^{(2)}} \quad . \quad . \quad . \quad (13)$$

and

$$\alpha^{(2)} = \alpha^{(1)} + \frac{\chi^{(2)} - \chi^{(1)}}{kT}, \quad . \quad . \quad . \quad (14)$$

which, in combination with (7), give

$$\frac{c^{(2)}}{c^{(1)}} = \frac{q^{(2)}}{q^{(1)}}\left(\frac{m^{(2)}}{m^{(1)}}\right)^{\frac{3}{2}}e^{-(\chi^{(2)}-\chi^{(1)})/kT} \quad . \quad . \quad . \quad (15)$$

This is effectively the Boltzmann equation.

(*b*) As a second example we will derive the ionisation formula when the electrons alone are degenerate. In this case by (6′) approximately

$$c_e = \frac{2\pi q_e}{h^3}(2m_e kT)^{\frac{3}{2}}\tfrac{2}{3}(-\alpha_e)^{\frac{3}{2}} \quad . \quad . \quad . \quad (9')$$

Hence, by (7), (8), (9′), and (11),

$$\log\left(\frac{c^{(1)}}{c_+^{(1)}}\,\frac{q_+^{(1)}}{q^{(1)}}\right)+\tfrac{3}{2}\log\frac{m_+^{(1)}}{m^{(1)}}=\frac{\chi}{k\mathrm{T}}+\frac{h^2}{2m_ek\mathrm{T}}\left(\frac{\frac{3}{2}c_e}{2\pi q_e}\right)^{\frac{2}{3}} \quad . \quad (12')$$

This is a simplified form of Chandrasekhar's formula. In it we may put $m_+^{(1)}/m^{(1)}=1$. The Boltzmann formula is unaffected.

(*c*) As a third example, let the atoms, ions, and electrons all be degenerate. Then (9′) holds for the electrons, and for the atoms and ions, by applying (6′),

$$c^{(1)}=\frac{2\pi q^{(1)}}{h^3}(2m^{(1)}k\mathrm{T})^{\frac{3}{2}}(-\alpha^{(1)})^{\frac{3}{2}} \quad . \quad . \quad . \quad (7')$$

$$c_+^{(1)}=\frac{2\pi q_+^{(1)}}{h^3}(2m_+^{(1)}k\mathrm{T})^{\frac{3}{2}}(-\alpha_+^{(1)})^{\frac{3}{2}} \quad . \quad . \quad (8')$$

By (7′), (8′), (9′), and (11)

$$\frac{h^2}{2k\mathrm{T}}\left[\frac{1}{m^{(1)}}\left(\frac{\frac{3}{2}c^{(1)}}{2\pi q^{(1)}}\right)^{\frac{2}{3}}-\frac{1}{m_+^{(1)}}\left(\frac{\frac{3}{2}c_+^{(1)}}{2\pi q_+^{(1)}}\right)^{\frac{2}{3}}\right]=\frac{\chi}{k\mathrm{T}}+\frac{h^2}{2k\mathrm{T}}\,\frac{1}{m_e}\left(\frac{\frac{3}{2}c_e}{2\pi q_e}\right)^{\frac{2}{3}} \quad (12'')$$

We notice the remarkable fact that T disappears from this equation, so that for complete degeneracy the degree of ionisation is ultimately independent of the temperature.

For degeneracy of the atoms and ions, the Boltzmann formula (15) will be found to be replaced by

$$\frac{h^2}{2k\mathrm{T}}\left[\frac{1}{m^{(1)}}\left(\frac{\frac{3}{2}c^{(1)}}{2\pi q^{(1)}}\right)^{\frac{2}{3}}-\frac{1}{m^{(2)}}\left(\frac{\frac{3}{2}c^{(2)}}{2\pi q^{(2)}}\right)^{\frac{2}{3}}\right]=\frac{\chi^{(2)}-\chi^{(1)}}{k\mathrm{T}} \quad . \quad (15')$$

from which T again disappears. Thus for complete degeneracy we have approximately

$$\left(\frac{c^{(1)}}{q^{(1)}}\right)^{\frac{2}{3}}-\left(\frac{c^{(2)}}{q^{(2)}}\right)^{\frac{2}{3}}=\text{const.}\times(\chi_2-\chi_1) \quad . \quad . \quad (15'')$$

5. *Proof of the Formulæ.*—A full discussion of the Fermi-Dirac statistics requires the use of the wave-mechanics. But for systems to which the Fermi-Dirac statistics are applicable the discussion may be replaced by the assumption of a certain principle, which is best enunciated at the point of the proof at which it is required. For simplicity, I propose to write out a proof for the particular case when the chemical equation is of the form

$$a\mathrm{A}+b\mathrm{B}=\mathrm{A}_a\mathrm{B}_b, \quad +\chi,$$

where a and b are integers and A, B, $\mathrm{A}_a\mathrm{B}_b$ are chemical symbols. Any other case is proved by similar arguments.

Suppose that the total number of A's present in volume V is X, that of B's, Y, and that these occur in the assembly as L free particles of type A, M free particles of type B, and N free particles of type $\mathrm{A}_a\mathrm{B}_b$. Then, by simple counting,

$$\mathrm{X}=\mathrm{L}+a\mathrm{N} \quad . \quad . \quad . \quad . \quad (16)$$

$$\mathrm{Y}=\mathrm{M}+b\mathrm{N} \quad . \quad . \quad . \quad . \quad (17)$$

Let the total kinetic energies be T_A, T_B, T_{AB}. Let q_A, q_B, q_{AB} be the statistical weights. The total energy U is given by

$$U = T_A + T_B + T_{AB} - N\chi,$$

reckoned from an obvious zero.

The phase-space available for the particles of type A with separate kinetic energies between E_s and E_{s+1}* is

$$\int \ldots . \int dxdydzdp_x dp_y dp_z,$$

where (x, p_x) (y, p_y) (z, p_z) are co-ordinates and conjugated momenta, the integral being taken over the volume V and over the momentum domain

$$E_s \leqslant \frac{p_x^2 + p_y^2 + p_z^2}{2m_A} \leqslant E_{s+1}.$$

The number of cells in this of volume h^3 is

$$\frac{V}{h^3} 4\pi r^2 dr,$$

where

$$r^2 = 2m_A E_s$$
$$dr = (2m_A)^{\frac{1}{2}} \tfrac{1}{2} E_s^{-\frac{1}{2}} dE_s. \qquad (dE_s = E_{s+1} - E_s).$$

This reduces to

$$\frac{2\pi V}{h^3} (2m_A)^{\frac{3}{2}} E_s^{\frac{1}{2}} dE_s.$$

If the weight is q^A, the effective number of cells is q_A times this. Call this number of cells x_s. Then

$$x_s = \frac{2\pi V q_A}{h^3} (2m_A)^{\frac{3}{2}} E_s^{\frac{1}{2}} dE_s \quad . \quad . \quad . \quad (18)$$

and similarly if y_s is the number of similarly defined cells for particles B, z_s for particles $A_a B_b$, we have

$$y_s = \frac{2\pi V q_B}{h^3} (2m_B)^{\frac{3}{2}} E_s^{\frac{1}{2}} dE_s \quad . \quad . \quad . \quad (18')$$

$$z_s = \frac{2\pi V q_{AB}}{h^3} (2m_{AB})^{\frac{3}{2}} E_s^{\frac{1}{2}} dE_s \quad . \quad . \quad . \quad (18'')$$

Into these x_s cells let us put p_s' particles of type A into separate cells, leaving p_s^0 cells unoccupied, where

$$p_s^0 + p_s' = x_s \quad . \quad . \quad . \quad . \quad (19)$$

The number of different ways of arranging the occupied and unoccupied cells in the phase space is

$$\frac{x_s!}{p_s^0!\, p_s'!}.$$

* The sequence (E_s) is an arbitrary partition of the kinetic energy range.

We have similarly that the number of ways of putting q_s' particles of type B into the y_s cells, leaving q_s^0 cells unoccupied, is

$$\frac{y_s!}{q_s^0!\, q_s'!},$$

where
$$q_s^0 + q_s' = y_s \quad . \quad . \quad . \quad . \quad (19')$$

and the number of ways of putting r_s' particles of type $A_a B_b$ into the z_s cells, leaving r_s^0 cells unoccupied, is

$$\frac{z_s!}{r_s^0!\, r_s'!},$$

where
$$r_s^0 + r_s' = z_s \quad . \quad . \quad . \quad . \quad (19'')$$

Then the *fundamental principle of the Fermi-Dirac statistics*, in conjunction with the *Boltzmann entropy principle*, asserts that *the average number of particles in the different types of cells will be those values of* p_s', q_s', r_s', *for which the entropy* S *given by*

$$S = k \log \prod_s \frac{x_s!}{p_s^0!\, p_s'!} \prod_s \frac{y_s!}{q_s^0!\, q_s'!} \prod_s \frac{z_s!}{r_s^0!\, r_s'!} \quad . \quad . \quad (20)$$

is a maximum subject to conditions (16), (17), (19), (19'), (19'') *and to the* further conditions

$$\sum_s p_s' = L, \qquad \sum_s q_s' = M, \qquad \sum_s r_s' = N \quad . \quad (21)$$

$$U = \sum_s p_s' E_s^A + \sum_s q_s' E_s^B + \sum r_s' (E_s^{AB} - \chi) \quad . \quad (22)$$

We have added indices to the E-symbols for clarity.
In the variation x_s, y_s, and z_s are kept fixed by (18), (18'), (18'').

Applying Stirling's theorem in the form

$$\log n! \sim n \log n - n,$$

and effecting a variation of the p's, q's, and r's, we find easily

$$0 = -\Sigma \delta p_s^0 (1 + \log p_s^0) - \Sigma \delta p_s' (1 + \log p_s') - \ldots \text{(terms in } q, r) \quad (23)$$

for all $\delta p_s^0 \ldots \delta r_s'$ subject to

$$0 = \delta p_s^0 + \delta p_s' \quad . \quad . \quad . \quad . \quad (24)$$

$$0 = \delta q_s^0 + \delta q_s' \quad . \quad . \quad . \quad . \quad (24')$$

$$0 = \delta r_s'^0 + \delta r_s' \quad . \quad . \quad . \quad . \quad (24'')$$

$$0 = \sum_s \delta p_s' + a \sum_s \delta r_s' \quad . \quad . \quad . \quad (25)$$

$$0 = \sum_s \delta q_s' + b \sum_s \delta r_s' \quad . \quad . \quad . \quad (25')$$

$$0 = \sum_s E_s^A \delta p_s' + \sum_s E_s^B \delta q_s' + \sum_s (E_s^{AB} - \chi) \delta r_s' \quad . \quad (26)$$

Associate with equations (24), (24′), (24″) undetermined multipliers $F_s + 1$, $G_s + 1$, $H_s + 1$, with equations (25) and (25′) multipliers α and β, and with (26) the multiplier γ. Combining the complete set of equations by Lagrange's method and equating to zero the coefficients of $\delta p_s^0 \ldots \delta r_s'$, we find

$$-\log p_s^0 + F_s = 0, \quad -\log q_s^0 + G_s = 0, \quad -\log r_s^0 + H_s = 0,$$
$$-\log p_s' + F_s - \alpha - \gamma E_s^A = 0, \quad -\log q_s' + G_s - \beta - \gamma E_s^B = 0,$$
$$-\log r_s' + H_s - a\alpha - b\beta - \gamma(E_s^{AB} - \chi) = 0.$$

Altering the constants F_s, G_s, H_s to new constants C_s^A, etc., we find from these

$$p_s^0 = C_s^A, \qquad p_s' = C_s^A e^{-(\alpha+\gamma E_s^A)}$$

whence

$$x_s = p_s^0 + p_s' = C_s^A(1 + e^{-(\alpha+\gamma E_s^A)})$$

This last determines C_s^A, whence

$$p_s' = x_s \frac{e^{-(\alpha+\gamma E_s^A)}}{1 + e^{-(\alpha+\gamma E_s^A)}} = \frac{x_s}{e^{\alpha+\gamma E_s^A} + 1} \quad . \quad . \quad (27)$$

and similarly

$$q_s' = \frac{y_s}{e^{\beta+\gamma E_s^B} + 1} \quad . \quad . \quad . \quad . \quad . \quad (27')$$

$$r_s' = \frac{z_s}{e^{a\alpha+b\beta+\gamma(E_s^{AB}-\chi)} + 1} \quad . \quad . \quad . \quad (27'')$$

The values of L, M, and N follow from (21). Replacing the sums by integrals, introducing the values of x_s, y_s, and z_s, and suppressing the suffixes to E we have

$$L = \frac{2\pi V q_A}{h^3}(2m_A)^{\frac{3}{2}} \int_0^\infty \frac{E^{\frac{1}{2}} dE}{e^{\alpha+\gamma E}+1} \quad . \quad . \quad . \quad . \quad (28)$$

$$M = \frac{2\pi V q_B}{h^3}(2m_B)^{\frac{3}{2}} \int_0^\infty \frac{E^{\frac{1}{2}} dE}{e^{\beta+\gamma E}+1} \quad . \quad . \quad . \quad . \quad (28')$$

$$N = \frac{2\pi V q_{AB}}{h^3}(2m_{AB})^{\frac{3}{2}} \int_0^\infty \frac{E^{\frac{1}{2}} dE}{e^{a\alpha+b\beta+\gamma(E-\chi)}+1} \quad . \quad . \quad (28'')$$

It is proved below that $\gamma = 1/kT$. Assuming this for the moment and writing in each integral $E = kTx$, we obtain the formulæ of paragraph 2, on putting

$$\alpha' = a\alpha + b\beta - \frac{\chi}{kT},$$

which being equivalent to

$$a\alpha + b\beta = \alpha' + \frac{\chi}{kT} \quad . \quad . \quad . \quad . \quad (29)$$

is simply formula (3) for the problem we are discussing. This proves the theorem.

6. To establish $\gamma = 1/kT$, substitute the values of $p_s^0 \ldots r_s^1$ already found into expression (20) for S and apply Stirling's theorem. After some lines of analysis it will be found that S is given by

$$\frac{S}{k} = \frac{2\pi V}{h^3}\Big[q_A(2m_A)^{\frac{3}{2}}\int_0^\infty \log(1+e^{-(\alpha+\gamma E)})E^{\frac{1}{2}}dE + q_B(2m_B)^{\frac{3}{2}}\int_0^\infty \log(1+e^{-(\beta+\gamma E)})E^{\frac{1}{2}}dE$$
$$+ q_{AB}(2m_{AB})^{\frac{3}{2}}\int_0^\infty \log(1+e^{-(a\alpha+b\beta+\gamma E-\gamma\chi)})E^{\frac{1}{2}}dE$$
$$+ L\alpha + M\beta + N(a\alpha+b\beta) + \gamma U \quad . \quad . \quad . \quad (30)$$

whilst U will be found to reduce to

$$U = \frac{2\pi V}{h^3}\Big[q_A(2m_A)^{\frac{3}{2}}\int_0^\infty \frac{E^{\frac{3}{2}}dE}{e^{\alpha+\gamma E}+1} + q_B(2m_B)^{\frac{3}{2}}\int_0^\infty \frac{E^{\frac{3}{2}}dE}{e^{\beta+\gamma E}+1}$$
$$+ q_{AB}(2m_{AE})^{\frac{3}{2}}\int_0^\infty \frac{E^{\frac{1}{2}}(E-\chi)dE}{e^{a\alpha+b\beta+\gamma E-\gamma\chi}+1}\Big], \quad (31)$$

or say,

$$U = U_A + U_B + U_{AB}.$$

If we take the differential of S keeping V constant we find

$$\frac{dS}{k} = -(Ld\alpha + U_A d\gamma) - (Md\beta + U_B d\gamma) - [N(ad\alpha + bd\beta) + U_{AB}d\gamma]$$
$$+ d(L\alpha) + d(M\beta) + d[N(a\alpha+b\beta)] + d(\gamma U)$$
$$= \gamma dU + \alpha dL + \beta dM + (a\alpha+b\beta)dN \quad . \quad (32)$$

Now by Willard Gibbs's thermodynamic definitions of partial potentials, μ_r per particle, we have at constant volume

$$dU = TdS + \sum_r \mu_r dL_r,$$

or in our case

$$dU = TdS + \mu_A dL + \mu_B dM + \mu_{AB}dN \quad . \quad . \quad (33)$$

Equations (32) and (33) must be identical for statistical mechanics to be consistent with thermodynamics. The conditions for this are

$$\gamma = \frac{1}{kT}, \quad \mu_A = -\alpha kT, \quad \mu_B = -\beta kT, \quad \mu_{AB} = -(a\alpha+b\beta)kT \quad (34)$$

This identifies γ, and shows further that Willard Gibbs's condition of chemical equilibrium

$$a\mu_A + b\mu_B = \mu_{AB} \quad . \quad . \quad . \quad . \quad (35)$$

is identically satisfied.

7. *Pressure.*—It is worth completing the investigation by determining the pressure. By integrating by parts the integrals occurring in the expression (30) for S, we find easily that

$$\frac{S}{k} = \tfrac{2}{3}\gamma(T_A + T_B + T_{AB}) + L\alpha + M\beta + N(a\alpha+b\beta) + \gamma U \quad (36)$$

where T_A, T_B, T_{AB} are the *kinetic* energies given by

$$T_A = U_A, \qquad T_B = U_B, \qquad T_{AB} = U_{AB} + N\chi,$$

so that

$$U = T_A + T_B + T_{AB} - N\chi.$$

This may be written, on putting $\gamma = 1/kT$,

$$S - \frac{U}{T} - \frac{2}{3}\frac{T_A + T_B + T_{AB}}{T} = k[L\alpha + M\beta + N(a\alpha + b\beta)]$$
$$= -\frac{1}{T}[L\mu_A + M\mu_B + N\mu_{AB}]$$

or

$$U - ST + \tfrac{2}{3}(T_A + T_B + T_{AB}) = L\mu_A + M\mu_B + N\mu_{AB} \quad . \qquad (37)$$

By the general theory of Willard Gibbs it follows that if p is the pressure, V the volume, then

$$pV = \tfrac{2}{3}(T_A + T_B + T_{AB}) \quad . \quad . \quad . \qquad (38)$$

so that

$$\tfrac{3}{2}p = \frac{2\pi kT}{h^3}\Big[q_A(2m_A kT)^{\frac{3}{2}}\int_0^\infty \frac{x^{\frac{3}{2}}dx}{e^{\alpha+x}+1} + q_B(2m_B kT)^{\frac{3}{2}}\int_0^\infty \frac{x^{\frac{3}{2}}dx}{e^{\beta+x}+1}$$
$$+ q_{AB}(2m_{AB}kT)^{\frac{3}{2}}\int_0^\infty \frac{x^{\frac{3}{2}}dx}{e^{a\alpha+b\beta-\chi/kT+x}+1}\Big] \qquad (39)$$

This or a similar formula evaluates the total pressure of any gas mixture.

8. On a future occasion I hope to use the foregoing formulæ to determine the condition of equilibrium between matter and radiation at temperatures so high that the reversible transformation of matter into radiation can occur, following Jeans. If P denotes a proton, E an electron, the chemical equation describing the annihilation of a proton and an electron to form radiation is

$$P + E = 0, \qquad + (m_p + m_e)c^2.$$

The formulæ of § 2 are then immediately applicable to the problem of determining the density of matter in equilibrium with radiation at any given temperature. But it would make the present paper too long to give the investigation here. The results have an important bearing on conditions in the interiors of stars.

1930 *August.*

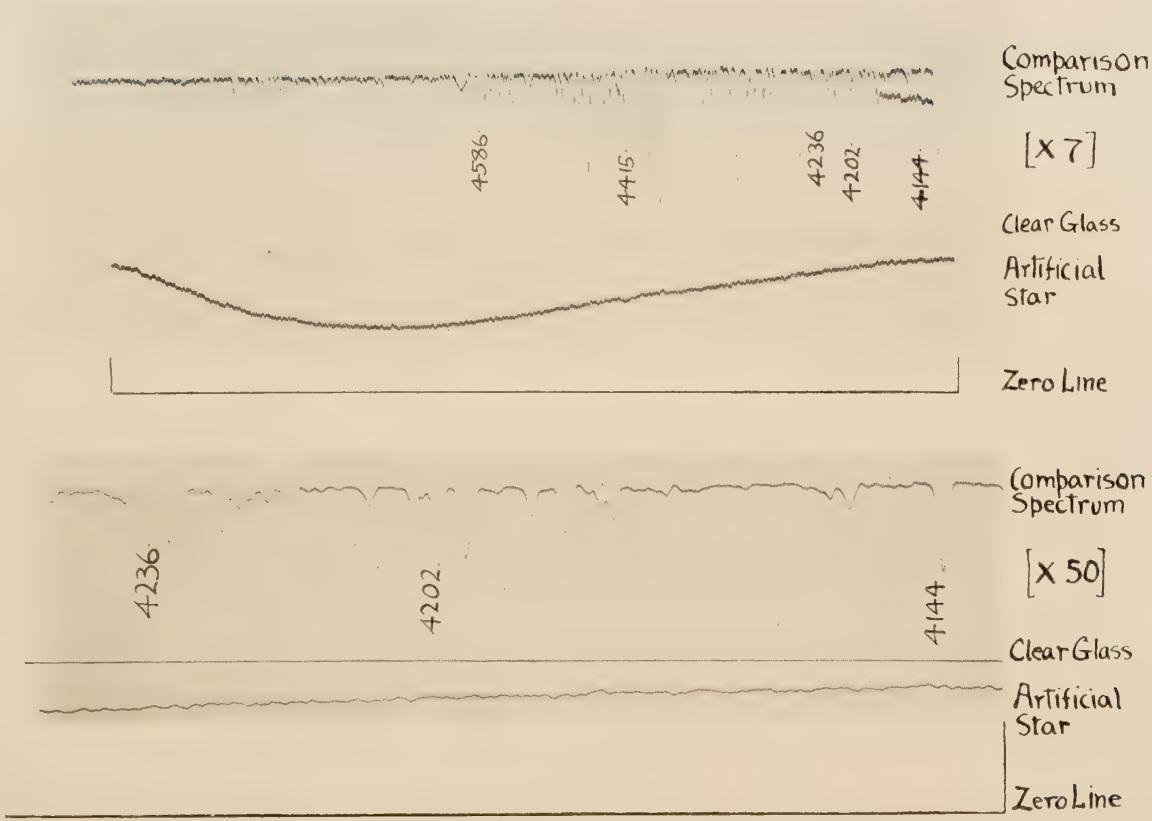

Fig. 1.—Transmission of light by optical system of 15-inch refractor of the Dominion Observatory.

Interlocking of Lines in Absorption Spectra.

By R. v. d. R. Woolley, B.A.

§ 1. In a previous paper * the present writer has discussed a formula due to H. A. Wilson † for the coefficient of line absorption in the formation of solar and stellar absorption lines, defined as follows. Let l_ν be the coefficient of line absorption: then if a beam of monochromatic light of frequency ν traverses unit length of an atmosphere which has a density ρ, a fraction ρl_ν of the energy is absorbed (and of course re-emitted. The process is sometimes called scattering). H. A. Wilson's formula for l_ν as a function of ν is

$$l_\nu = \frac{l_0}{1 + a^2(\nu - \nu_0)^2},$$

where $l_0 (= l_{\nu_0})$ and a are constants for any particular absorption line. In the paper mentioned above the writer found an expression for the coefficient in the case of an assembly of atoms having a Maxwellian distribution of velocity corresponding to a temperature T, a modification being due to the Doppler effect. If l_ν' is the modified coefficient, we have (*loc. cit.*, equation (3·3)) (writing l_ν for s_ν)

$$l_\nu' = l_0 \frac{1}{\sqrt{\pi}} \left(\frac{M}{2RT}\right)^{\frac{1}{2}} \frac{c}{a\nu} \int_0^\infty u e^{-\frac{Mu^2}{2RT}} \left[\tan^{-1} a\left(\nu - \nu_0 + \frac{\nu u}{c}\right) - \tan^{-1} a\left(\nu - \nu_0 - \frac{\nu u}{c}\right)\right] du,$$

where M is the mass of each atom and R is Boltzmann's constant. Using the values given in *loc. cit.* for l_0 and a, we find the following values of l_ν' for the Ca^+ line K :—

Table I.

$\frac{\nu - \nu_0}{\nu_0}$	0	3·3 10^{-6}	6·6 10^{-6}	10^{-5}	2·10^{-5}
l_ν	7·4 10^{12}	1·54 10^8	3·85 10^7	1·72 10^7	4·30 10^6
l_ν'	1·7 10^{10}	1·2 10^{10}	3·4 10^9	4·6 10^8	4·8 10^6

For higher values of $\left|\frac{\nu - \nu_0}{\nu_0}\right|$, $l_\nu' = l_\nu$.

§ 2. We can insert these values of l_ν' into the formula for the contour of an absorption line, and find the blackening $r(\nu)$ as a function of the frequency ν. $r(\nu)$ is the ratio of the intensity of light emitted between frequencies ν to $\nu + d\nu$ to the intensity in an equal range of frequency in the background just outside the line.

Substituting in Eddington's formula for $r(\nu)$, ‡

$$r(\nu) = \frac{b + aq}{b + a\sqrt{3}} \cdot \frac{1 + \frac{2}{3}\sqrt{3}}{1 + \eta + \frac{2}{3}q}, \qquad (2.0)$$

* *M.N.*, **90**, 170, 1929.
† H. A. Wilson, *Proc. Roy. Soc.*, **118**, 362, 1928.
‡ *M.N.*, **89**, 625, 1929.

where a and b are constants,

$\eta = l_\nu/k$,
$q^2 = 3(1+\eta)(1+\epsilon\eta)$,
$\epsilon =$ the fraction of the energy absorbed by superelastic collisions,
$k =$ the coefficient of continuous absorption.

We have * for Ca^+ K, $b/a = 2{\cdot}25$, $k = 338$, and $\epsilon = 10^{-3}$. Then from Table I. we have $\eta(\nu_0) = 5{\cdot}0\ 10^7$ and $\eta(\nu_1) = 1{\cdot}5\ 10^4$ where $\left|\frac{\nu_1-\nu_0}{\nu_0}\right| = 2{\cdot}10^{-5}$. Then from (2·0) $r(\nu_0) = r(\nu_1)$ to within about 1 part in 10^3. The contour is flat at the centre, and an atom above Eddington's model absorbs from the flat centre of the line and not from the wings of the line. To see this, consider the amount of momentum absorbed per second by an atom from uniform intensity I in the range $\nu_1 \geqslant \nu \geqslant \nu_0$, which is

$$\frac{M}{c}\int_{\nu_0}^{\nu_1} I l_\nu d\nu = \frac{M l_0}{a}\frac{I}{c}\tan^{-1} a(\nu-\nu_0).$$

Since a is of order 10^{-9}, more than 99·9 per cent. is absorbed in the range $|\nu-\nu_0| < 1{\cdot}5\ 10^{10}$, which is the extent of the flat part of the contour. (As an example, if we suppose that $r(\nu) = 1$ for $|\nu-\nu_0| > 1{\cdot}5\ 10^{10}$ and $r(\nu) = 0{\cdot}01$ for $|\nu-\nu_0| < 1{\cdot}5\ 10^{10}$, even in these unfavourable circumstances more than 95 per cent. is absorbed from the centre.)

Thus in the case of a broad line such as K, the process, sometimes suggested as a possibility, that there may be a transference of energy from the wings to the centre of the line, is not likely to have a great effect on the central intensity. This remark is not intended as a rigorous treatment of the question of redistribution in frequency, which (*vide* Eddington, *M.N.*, **89**, 621, 1929) we have tacitly assumed not to occur. The question is, when an atom absorbs a quantum of frequency $\nu_0 + d\nu$, does it (as we have assumed) necessarily emit a quantum of exactly the same frequency? It has been suggested that an atom in the outer layers of the atmosphere will absorb from the wings of the line, being starved of light of the central frequency owing to the blackening of the line, and will then switch the radiation into the central frequency. But we have shown that over Eddington's model the atom does not absorb a large fraction of the energy from the wings of the line. If we regard Eddington's model as an approximation to the new model in which redistribution is allowed, the effect of redistribution on the central intensity is negligible because the absorption in the wings is small.

We can find the effect on the wings in the following manner: the ordinary solution for the central intensity is taken unaltered. We suppose that the re-emission in any frequency depends only on the integrated amount of energy absorbed, and that the amount of energy emitted between frequencies ν and $\nu + d\nu$ is proportional to $l_\nu d\nu$ (this

* Woolley, *M.N.*, **90**, 180, 1929.

is the extreme case of *complete* redistribution). The equation of transfer is, in Eddington's notation,*

$$\cos\theta\frac{dJ_\nu'(\theta)}{\rho dr} = -(k+l_\nu)J_\nu'(\theta) + (1-\epsilon)l_\nu J_{\nu_0} + \epsilon l_\nu J_T + kJ_T,$$

where †

$$J_{\nu_0} = J + Ae^{-q_0\tau}$$

(we take the case $J = J_T$) and

$$A = -\frac{2H\eta_0}{1+\eta_0+\frac{2}{3}q_0}.$$

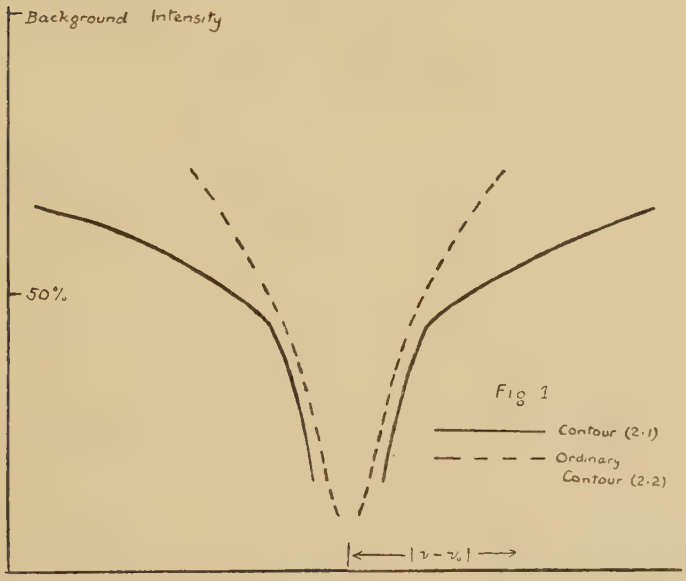

Fig. 1.—Comparison of (2.1) (from Table II.) with ordinary contour, fitted approximately at $r=\frac{1}{2}$, showing "wings" and "Y shape" of (2.1).

The boundary conditions are the usual

$$J_\nu' \to J_T \quad \text{as} \quad \tau \to \infty \quad \text{and} \quad J_\nu' = 2H_\nu' \quad \text{at} \quad \tau = 0.$$

We find

$$r(\nu) = \frac{1+\frac{2}{3}p}{1+\eta+\frac{2}{3}p} - \frac{r^2}{p+q_0}\frac{\frac{2}{3}\eta_0}{(1+\eta+\frac{2}{3}p)(1+\eta_0+\frac{2}{3}q_0)} \qquad (2.1)$$

where

$$p^2 = 3(1+\eta_\nu)^2,$$
$$q_0^2 = 3(1+\eta_0)(1+\epsilon\eta_0),$$
$$r^2 = 3(1-\epsilon)\eta_\nu(1+\eta_\nu).$$

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* *M.N.*, **89**, 622, 1929. † *Ibid.*, p. 624, equation (14).

In the wings of the line the second term is small and we have

$$r(\nu) = \frac{1 + \frac{2}{3}p}{1 + \eta + \frac{2}{3}p},$$

the second term producing a central darkening, the whole contour being "Y-shaped." For a given half-width $|\nu - \nu_0|$ the contour (2·1) gives a higher value of η than the ordinary contour, and thus a lower value of K (about one quarter), but there is a marked difference in the two shapes. Table II. shows values of $r(\nu)$ from (2·1) and from the Eddington formula,

$$r(\nu) = \frac{1 + \frac{2}{3}q}{1 + \eta + \frac{2}{3}q}, \qquad (2.2)$$

with

$$q^2 = 3(1 + \eta)(1 + \epsilon\eta).$$

Here

$$\epsilon = 10^{-3}.$$

The values of $|\nu - \nu_0|$ are in arbitrary units (depending on the value of k).

Table II.

$\nu - \nu_0$	10	7	5·8	4	3	1·4	1	0·7
η	1	2	3	6	10	50	100	200
$r(\nu)$ from (2·1)	0·76	·69	·65	·60	·56	·45	·35	·15
$r(\nu)$ from (2·2)	0·72	·60	·52	·41	·33	·16	·11	·09

§ 3. *Interlocking of Lines.*—As was pointed out by Eddington,* the ordinary theory of the formation of absorption lines applies only to principal lines. It is known from experiment and it is in accordance with atomic theory that, in the case of other lines, when an atom performs selective absorption, the corresponding emission is not confined to the frequency of absorption, but may take place in a number of frequencies characteristic of the element under consideration. The phenomenon is best illustrated by direct reference to atomic theory. We take the simplest case, that of the hydrogen line $H\alpha$. Absorption of $H\alpha$ radiation occurs when an excited H atom whose electron is in state 2 moves to state 3. The emission of $H\alpha$ occurs when an electron in state 3 moves to state 2, but the electron in state 3 can also move to state 1, in which case it emits light of a different frequency L_β. In the absence of other stimulus, an (excited) hydrogen atom exposed to $H\alpha$ light absorbs it, and may emit either $H\alpha$ or L_β.

In the case of the principal line L_α the electron moves from state 1 to state 2 in absorption : and it can only re-emit by moving back from 2 to 1 and emitting the same frequency L_α.

In thermodynamical equilibrium, the atom will absorb energy from $H\alpha$ and from L_β and emit both. There is detailed balancing of these processes, so that the total emission of energy in the $H\alpha$ frequency

* *M.N.*, **89**, 621, 1929.

is equal to the total absorption in the same frequency. But since the formation of an absorption line in an atmosphere is a consequence of departure from thermodynamical equilibrium, it is obvious that we cannot consider the formation of any absorption line without conducting a simultaneous discussion of the formation of all the lines with which it is interlocked. In the case of principal lines it is assumed that there is detailed balancing of the higher transitions, *i.e.* in the case of the line L_α, the number of atoms leaving state 2 by absorbing in the Balmer series is balanced by the number returning to state 2 by emission of the same series. This approximation is justified by the fact that the transition 2–1 is much more probable than any other transition from state 2.

We shall attempt to solve the problem of two interlocked lines in a model atmosphere (specifically $H\alpha$ and L_β) by a similar method—that is to say, by assuming that there is detailed balancing of the atomic processes involving transitions from state 3 to higher states

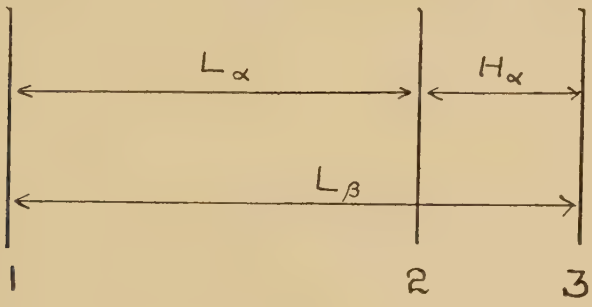


Fig. 2.

(the Paschen series). These transitions are again much less probable than the transitions 3–2 and 3–1. We have to make an assumption as to the number of atoms in state 2. This will be chiefly controlled by the powerful L_α line, and will be proportional to the central intensity J_{ν_0}' of that line. As we go downwards from the surface this intensity becomes sensibly equal to the equilibrium value J very quickly (much more quickly than the outflow H_ν' tends to H). It is consistent with the other approximation (η constant) in Eddington's atmosphere, though not, of course, accurate, to take the ratio of the number of atoms in state 2 to the number in state 1 as being constant throughout the atmosphere.

We suppose that when an atom absorbs a quantum of frequency $\nu_\alpha + d\nu$, the corresponding emission is one of two frequencies $\nu_\alpha + d\nu$ and $\nu_\beta + d\nu$. The two frequencies $\nu_\alpha + d\nu$ and $\nu_\beta + d\nu$ are interlocked with each other and with no other frequencies.

§ 4. Consider the transfer of energy connected with two interlocked lines whose central frequencies are ν_α and ν_β. The ν_α radiation will lose energy by selective and by continuous absorption. There will be re-emission of a fraction $(1 - \epsilon)$ of the selected radiation, and a fraction

ϵ is lost by superelastic collisions. The corresponding emission due to inelastic collisions is $\epsilon l_\alpha J_{(\alpha T)}$ per gram, where $J_{(\alpha T)}$ is the equilibrium value of J_α when in thermodynamical equilibrium with matter at temperature T, the temperature of the atmosphere. Of the fraction $(1 - \epsilon)$ of the selectively absorbed radiation, a fraction λ_α appears as ν_α radiation and a fraction λ_β is switched into the frequency ν_β by the interlocking process. Here $\lambda_\alpha + \lambda_\beta = 1$. Consideration of detailed balancing under thermodynamical equilibrium brings to light relations between the λ's and other atomic constants.

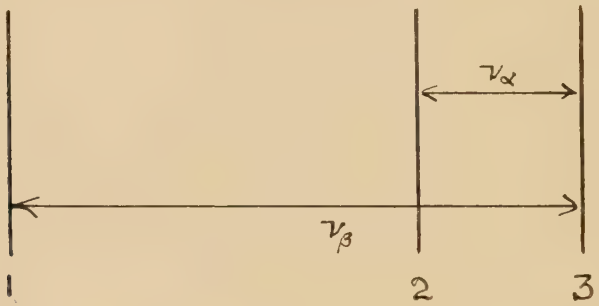


Fig. 3.

Let n_1 be the number of atoms per c.c. in state 1,
n_2 be the number of atoms per c.c. in state 2,
N_{13} the number of transitions per c.c. per sec. from state 1 to state 3,
N_{23} the number of transitions per c.c. per sec. from state 2 to state 3,
p_α the fraction of the atoms leaving state 3 per c.c. per sec. which descend to state 2,
p_β the fraction of the atoms leaving state 3 per c.c. per sec. which descend to state 1,

so that $p_\alpha + p_\beta = 1$. Since the energy corresponding to the two transitions is $h\nu_\alpha$ and $h\nu_\beta$ respectively, then

$$\frac{\lambda_\alpha}{\lambda_\beta} = \frac{p_\alpha \nu_\alpha}{p_\beta \nu_\beta}.$$

The number of atoms arriving at state 3 per sec. is $N_{13} + N_{23}$, which must be equal to the number leaving, so that we have

$$N_{23} = N_{32} = p_\alpha(N_{31} + N_{32}), \quad \text{or} \quad \frac{p_\alpha}{N_{23}} = \frac{p_\beta}{N_{13}}.$$

We have for the energy absorbed per c.c. per sec. in the frequency ν_α,

$$N_{23} h\nu_\alpha = n_2 l_\alpha J_{(\alpha T)},$$

and in the frequency ν_β,

$$N_{13} h\nu_\beta = n_1 l_\beta J_{(\beta T)}.$$

But (in the Boltzmann statistics)

$$\frac{n_2}{n_1} = \frac{q_2}{q_1} e^{-\frac{h(\nu_\beta - \nu_\alpha)}{RT}},$$

where q_1, q_2 are the weights of the states 1 and 2.

Also

$$\frac{J_{(\beta T)}}{J_{(\alpha T)}} = \frac{\nu_\beta^3}{\nu_\alpha^3} e^{-\frac{h(\nu_\beta - \nu_\alpha)}{RT}},$$

so that

$$\frac{n_2}{n_1} = \frac{q_2}{q_1} \frac{\nu_\alpha^3}{\nu_\beta^3} \frac{J_{(\beta T)}}{J_{(\alpha T)}}.$$

We must therefore have

$$\frac{\lambda_\alpha}{q_2 l_\alpha \nu_\alpha^3} = \frac{\lambda_\beta}{q_1 l_\beta \nu_\beta^3} \quad . \quad . \quad . \quad . \quad (4.1)$$

Similar results hold for more complicated systems of interlocking, *e.g.* for the Paschen α line, H_β in the Balmer series, and Lyman γ we have

$$\frac{\lambda_\alpha}{q_3 l_\alpha \nu_\alpha^3} = \frac{\lambda_2}{q_2 l_\beta \nu_\beta^3} = \frac{\lambda_\gamma}{q_1 l_\gamma \nu_\gamma^3}.$$

§ 5. We consider together the two interlocked frequencies ν_1 and ν_2, where $\nu_1 = \nu_\alpha + d\nu$ and $\nu_2 = \nu_\beta + d\nu$.

We have already enumerated in § 4 all the gains and losses experienced by the ν_1 radiation except the amount switched into the frequency from ν_2, which is $\lambda_\alpha(1 - \epsilon) n_1 M l_2 J_{\nu_2}'$ units per c.c. per sec., where M is the mass of the atom. We are therefore led to the following simultaneous differential equations, with the usual notation:

$$\begin{aligned}
\cos\theta \frac{dJ_{\nu_1}'(\theta)}{dr} &= -(k\rho + n_2 M l_1) J_{\nu_1}'(\theta) + (1-\epsilon)\lambda_\alpha n_2 M l_1 J_{\nu_1}' \\
&\quad + (k\rho + \epsilon n_2 M l_1) J_{(\nu_1 T)} + (1-\epsilon)\lambda_\alpha n_1 M l_2 J_{\nu_2}' \\
\cos\theta \frac{dJ_{\nu_2}'(\theta)}{dr} &= -(k\rho + n_1 M l_2) J_{\nu_2}'(\theta) + (1-\epsilon)\lambda_\beta n_1 M l_2 J_{\nu_2}' \\
&\quad + (k\rho + \epsilon n_1 M l_2) J_{(\nu_2 T)} + (1-\epsilon)\lambda_\beta n_2 M l_1 J_{\nu_2}' \quad (5.1)
\end{aligned}$$

Writing

$$\frac{n_2 M l_1}{k\rho} = \eta_1, \quad \frac{n_1 M l_2}{k\rho} = \eta_2, \quad \text{and} \quad \tau = -\int k\rho dx,$$

and proceeding in the usual way, we get

$$\begin{aligned}
\frac{d^2 J_{\nu_1}'}{d\tau^2} &= 3(1+\eta_1)[J_{\nu_1}'(1 + \lambda_\beta\eta_1 + \epsilon\lambda_\alpha\eta_1) - (1+\epsilon n_1) J_{(\nu_1 T)} - (1-\epsilon)\lambda_\alpha\eta_2 J_{\nu_2}'] \\
\frac{d^2 J_{\nu_2}'}{d\tau^2} &= 3(1+\eta_2)[J_{\nu_2}'(1 + \lambda_\alpha\eta_2 + \epsilon\lambda_\beta\eta_2) - (1+\epsilon\eta_2) J_{(\nu_2 T)} - (1-\epsilon)\lambda_\beta\eta_1 J_{\nu_1}'] \quad (5.2)
\end{aligned}$$

with

$$H_{\nu_1}' = \frac{1}{3(1+\eta_1)} \frac{dJ_{\nu_1}'}{d\tau}, \qquad H_{\nu_2}' = \frac{1}{3(1+\eta_2)} \frac{dJ_{\nu_2}'}{d\tau}.$$

We have supposed $$J_{(\nu_1 T)} = a_1 + b_1\tau,$$
$$J_{(\nu_2 T)} = a_2 + b_2\tau.$$

Write (5·2) in the form

$$\frac{d^2 J_{\nu_1}'}{d\tau^2} = p^2 J_{\nu_1}' - q^2 J_{(\nu_1 T)} - r^2 J_{\nu_2}', \quad . \quad . \quad (5.3)$$

$$\frac{d^2 J_{\nu_2}'}{d\tau^2} = s^2 J_{\nu_2}' - u^2 J_{(\nu_2 T)} - t^2 J_{\nu_1}'.$$

We get, assuming η_1, η_2 and ϵ constant,

$$J_{\nu_1}' = A_1 e^{\lambda_1\tau} + B_1 e^{\lambda_2\tau} + C_1 e^{\lambda_3\tau} + D_1 e^{\lambda_4\tau} + \frac{s^2q^2}{s^2p^2 - r^2t^2} J_{(\nu_1 T)} + \frac{r^2u^2}{s^2p^2 - r^2t^2} J_{(\nu_2 T)}$$

$$J_{\nu_2}' = A_2 e^{\lambda_1\tau} + B_2 e^{\lambda_2\tau} + C_2 e^{\lambda_3\tau} + D_2 e^{\lambda_4\tau} + \frac{t^2q^2}{s^2p^2 - r^2t^2} J_{(\nu_1 T)} + \frac{p^2u^2}{s^2p^2 - r^2t^2} J_{(\nu_2 T)} \quad (5$$

where λ_1, λ_2, λ_3, λ_4 are the roots of the equation

$$(x^2 - p^2)(x^2 - s^2) - r^2t^2 = 0 \quad . \quad . \quad . \quad (5.5)$$

There are four relations $(s^2 - \lambda_1^2)A_2 = t^2 A_1$, etc., between the eight constants and four boundary conditions, viz. at $\tau = \infty$, $J_{\nu_1}' = J_{(\nu_1 T)}$, and $J_{\nu_2}' = J_{(\nu_2 T)}$: and at $\tau = 0$, $J_{\nu_1}' = 2H_{\nu_1}'$, and $J_{\nu_2}' = 2H_{\nu_2}'$. The solution of (5·5) is

$$x = \pm \sqrt{\{\tfrac{1}{2}(s^2 + p^2 \pm \sqrt{(s^2 - p^2)^2 + 4r^2t^2})\}}.$$

Since

$$4r^2t^2 < 4s^2p^2,$$

there are four real roots, two positive and two negative. The boundary conditions at infinity give that the coefficients of the exponentials with the positive roots are all zero.

It can be verified that

$$\frac{s^2q^2}{s^2p^2 - r^2t^2} J_{(\nu_1 T)} + \frac{r^2u^2}{s^2p^2 - r^2t^2} J_{(\nu_2 T)} = J_{(\nu_1 T)}$$

and

$$\frac{t^2q^2}{s^2p^2 - r^2t^2} J_{(\nu_1 T)} + \frac{p^2u^2}{s^2p^2 - r^2t^2} J_{(\nu_2 T)} = J_{(\nu_2 T)}$$

so that (5·4) becomes

$$\left.\begin{aligned} J_{\nu_1}' &= A_1 e^{\lambda_1\tau} + B_1 e^{\lambda_2\tau} + J_{(\nu_1 T)} \\ J_{\nu_2}' &= A_2 e^{\lambda_1\tau} + B_2 e^{\lambda_2\tau} + J_{(\nu_2 T)} \end{aligned}\right\} \quad . \quad . \quad (5.6)$$

The boundary conditions at $\tau = 0$ give the following equations :—

$$A_1(1 + \eta_1 - \tfrac{2}{3}\lambda_1) + B_1(1 + \eta_1 - \tfrac{2}{3}\lambda_2) + a_1(1 + \eta_1) - \tfrac{2}{3}b_1 = 0,$$
$$A_2(1 + \eta_2 - \tfrac{2}{3}\lambda_1) + B_2(1 + \eta_2 - \tfrac{2}{3}\lambda_2) + a_2(1 + \eta_2) - \tfrac{2}{3}b_2 = 0,$$

with

$$(s^2 - \lambda_1^2)A_2 = t^2 A_1,$$
$$(s^2 - \lambda_2^2)B_2 = t^2 B_1.$$

Now

$$\frac{b_1}{a_1} = \frac{3}{8}\frac{xe^2}{e^2-1} \quad \text{where} \quad x = \frac{h\nu_a}{\mathrm{RT}},$$

and

$$(\mathrm{H})_1 = \frac{b_1 + a_1\sqrt{3}}{3+2\sqrt{3}}.$$

Also

$$\frac{a_2}{a_1} = \frac{\nu_\beta^3}{\nu_a^3} e^{-\frac{h(\nu_\beta-\nu_a)}{\mathrm{RT}}}.$$

Hence the contour of the ν_a can be found directly.

$$\frac{\mathrm{H}_{\nu_1}'}{\mathrm{H}_1} = \frac{1}{3(1+\eta_1)}\left\{\lambda_1\frac{\mathrm{A}_1}{\mathrm{H}_1} + \lambda_2\frac{\mathrm{B}_1}{\mathrm{H}_1} + \frac{b_1}{\mathrm{H}_1}\right\}.$$

Similarly

$$\frac{\mathrm{H}_{\nu_2}'}{\mathrm{H}_2} = \frac{1}{3(1+\eta_2)}\left\{\lambda_1\frac{\mathrm{A}_2}{\mathrm{H}_2} + \lambda_2\frac{\mathrm{B}_2}{\mathrm{H}_2} + \frac{b_2}{\mathrm{H}_2}\right\} \quad . \quad . \quad (5.7)$$

It is not obvious that these equations give the same contours as if there had been no interlocking, but this is so, at any rate away from the centres of the lines.

For example, in the case of *Hα* and L_β, we have n_1/n_2 very large. In this case λ_1 is large compared with λ_2, but A_1 is so small compared with B_1 that $\lambda_1\mathrm{A}$ is small compared with $\lambda_2\mathrm{B}$. It can be verified that λ_2 reduces to $-\sqrt{3(1+\eta_1)}$, where n_2, η_1 refer to *Hα* and n_1, η_2 to L_β.

We then find for H_a,

$$r(\nu) = \frac{1-\frac{2}{3}\lambda_2}{1+\eta-\frac{2}{3}\lambda_2},$$

which is exactly the same result as for a principal line (with $\epsilon = 0$). It can be verified numerically that for a close multiplet (*i.e.* η_1 and η_2 of the same order of magnitude), the widths of the lines given by (5.7) do not differ appreciably from the widths given by the theory which neglects interlocking.

Conclusion.

The conclusion to be drawn from the equations is that, making the same approximations as are ordinarily made in the case of a principal line, the contour found for a line such as *Hα* has the same width as if no interlocking had taken place. This means that we can determine numbers of atoms in the ordinary way from *Hα* alone, and ignore the phenomenon of interlocking.

The physical cause of this seems to be the very rapid recovery of J_ν' to its equilibrium value J_T as we go beneath the surface. This recovery is more rapid in the case of the stronger line, and the interlocking effect is negligible.

Gonville and Caius College, Cambridge:
1930 *June* 12.

A Note on Unstable Layers in the Sun.

By Sir Gilbert T. Walker, C.S.I., Sc.D., F.R.S.

It was pointed out many years ago by Deslandres that the polygonal pattern, conspicuous in many spectroheliograms in K light, was presumably due to unstable layers ; for, as Bénard had shown, the equilibrium of a horizontal layer of fluid whose upper surface is made heavier than its lower surface breaks down in a polygonal pattern. Experiments made by S. Mal under my direction lead me to think that if the fluid is heated from below, the flow will be upwards within the polygons and downwards in the vertical current sheets between them ; and conversely, that if the instability is due to cooling of the top surface there will be downflow in the polygons and upflow between them. If ascending gases in the Sun are less bright than descending, the brightness of the polygonal boundaries, in ordinary photographs as well as in *K* and *Hα* spectroheliograms, would suggest that the instability was mainly due to heating from below.

It has been shown by Idrac that when there is sufficient shear occurring in the unstable layer there will develop straight vortices, alternatively right-handed and left-handed, with their axes along the lines of shear. In the Sun the ascent between an adjacent pair of these vortices and descent in the next gap will show themselves as darker and brighter lines. So we should expect a stream-line pattern where the horizontal shear is rapid enough ; and that will presumably be near a spot where the rate of change of horizontal velocity with height will be greatest. In fact we have the explanation of Hale's stream-lines in *Hα* spectroheliograms near spots.

If this interpretation is correct a further effect may be looked for. As S. Mal's experiments showed, when the shear is small the pattern produced in the unstable layer is rectangular with boundaries along and perpendicular to the shear. We could not be certain of finding this effect in the Sun, for the stream-lines occur only at *Hα* level, where the polygons are scarcely visible ; but we should hope to find it in the ends of the stream-lines furthest from the umbra, where the shear is diminishing ; and it is at times quite clearly visible, for example, in Plate 24 of Hale's recent account of the spectrohelioscope.*

In confirmation of the power of the instability to produce big scale effects it may be added that S. Mal was recently enabled by Professor Hergesell to make a number of measurements from an aeroplane, and found in every case that a stratified cloud, when broken up, was vertically unstable, and when unbroken, stable. Also long roll clouds corresponded to big differences of velocity above and below the sheets, while a polygonal pattern meant an inappreciable shear.

* *Mount Wilson Cont.*, 388, Plate 24, 1929, or *Ap. J.* **70**, 294, Plate 19, 1929.

The Solar Apex and Galactic Rotation. By Henri Mineur.

(*Communicated by the Secretaries.*)

I. *The Apex depends on the Distance of the Stars from the Galactic Plane.*

1. We shall denote by z the distance of a star from a plane parallel to the galactic plane and passing through the sun.

Our researches on the galactic rotation have led us to think that the mean velocity of the stars situated in a small volume of space depends chiefly on z.

The evidence of this phenomenon can be made clear by determining the motion of the sun with regard to groups of stars at various distances from the galactic plane. The proper motions being liable to present systematic errors modifying the position of the apex, we have obtained the determinations by means of the radial velocities only.

We have used for that purpose all the radial velocities now published (about 5500). We have grouped the stars into categories according to the values of z. Then using a method described in a memoir soon to be published in the *Bulletin de l'Observatoire de Lyon*, we have determined the apex and the velocity of the sun with regard to these various groups of stars. The results are in Table I., where X, Y, Z denote the components, in km./sec., of the solar velocity W with reference to galactic axes.

TABLE I.

z (parsecs).	Number of Stars.	X.	Y.	Probable Error.	Z.	Probable Error.	Apex. Galactic. Longitude.	Apex. Galactic. Latitude.	Sun's Velocity.	Apex. Equatorial Co-ordinates. A.	Apex. Equatorial Co-ordinates. D.
							°	°		°	°
−234	85	+13·22	+15·30	3·1	+ 2·71	1·6	+49·1	+ 8·0	20·42	299·5	+46·6
−196	251	+ 6·14	+13·61	2·3	+ 5·66	0·9	+65·7	+20·8	15·97	292·7	+66·8
−135	275	+ 7·21	+12·71	1·3	+ 6·46	0·9	+60·4	+23·9	15·98	282·1	+63·2
− 98	505	+11·43	+11·36	1·0	+ 7·12	0·7	+44·8	+23·5	17·58	275·7	+49·3
− 56	753	+14·34	+ 8·70	0·7	+ 6·76	0·7	+31·3	+21·9	18·09	272·5	+36·9
− 35	797	+16·84	+ 8·28	0·6	+ 6·76	0·9	+26·2	+19·8	19·95	272·9	+31·7
− 22	761	+14·57	+ 8·14	0·6	+ 9·18	1·0	+29·0	+28·7	19·12	263·9	+37·0
0	1654	+15·14	+ 6·81	0·4	+ 1·93	1·2	+24·2	+ 6·6	16·71	285·6	+24·5
+ 22	809	+15·45	+ 5·14	0·8	+ 7·01	0·8	+18·4	+23·3	17·73	276·3	+26·2
+ 36	899	+15·57	+ 7·64	0·6	+ 4·45	0·8	+26·1	+14·4	17·91	281·3	+29·6
+ 57	907	+17·14	+ 9·21	0·7	+ 5·55	0·6	+28·3	+15·9	20·23	278·1	+32·1
+ 97	773	+16·42	+ 3·88	0·9	+ 7·36	0·7	+13·3	+23·6	18·41	264·1	+22·0
+147	435	+15·99	+ 4·38	0·5	+ 8·11	9·7	+15·3	+26·1	18·45	262·3	+24·6
+190	369	+12·93	− 1·74	1·8	+ 8·12	0·7	− 7·7	+31·9	15·36	248·3	+ 8·5
+234	100	+ 9·85	− 2·83	2·5	+10·11	1·4	−16·1	+44·6	14·39	233·9	+ 9·0

It appears clearly that the apex does vary systematically with z. Several consequences can be deduced.

2. It has often been noticed that the declination D of the apex varies from + 20° to + 70° according to the stars which are considered, whilst the R.A. is less variable. This fact has been sometimes explained by the systematic errors of the proper motions in declination.

The above table shows that this variation of D is real, as the errors of the radial velocities cannot systematically affect the apex.

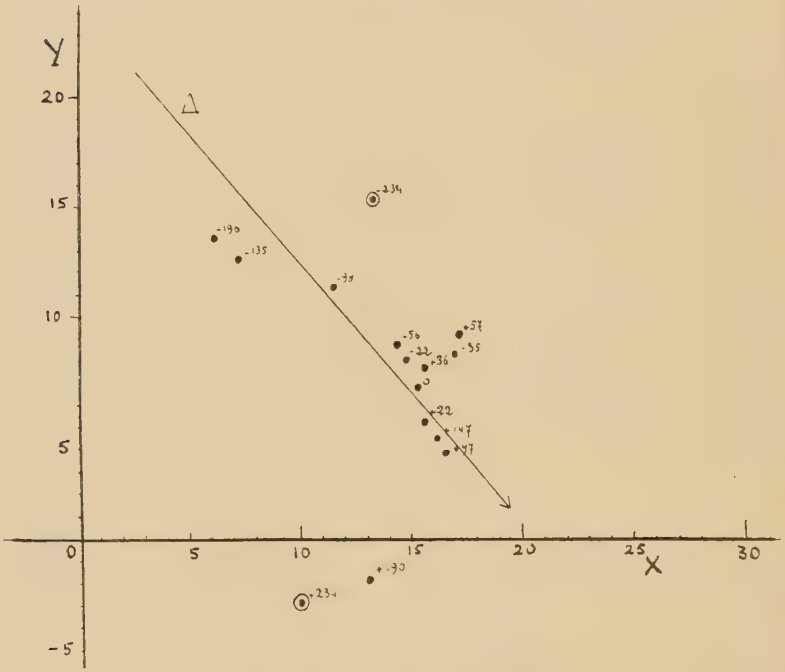


Fig. 1.—The projection on the galactic plane of the velocity vectors of the solar motion with respect to groups of stars at various distances (z) from the galactic plane.

It is of importance to pay regard to this result in the determination of statistical parallaxes, where the motion of the sun is used. Thus one might be led to revise the absolute magnitudes of the Cepheids and possibly other results.

3. Fig. 1 represents the extremities of the velocity vectors of the sun projected upon the galactic plane with reference to stars at various distances z from this plane. The numbers near the dots are the corresponding values of z. The dots with circles correspond to large values of z, *i.e.* to stars not belonging to the local system. The points have a clear tendency to place themselves on a straight line Δ and to move in the direction of the arrow as the value of z increases. The longitude

of the direction Δ is about 320°. This phenomenon is not unlike the asymmetrical drift of Strömberg; let us call it for the present the "second asymmetrical drift." Its importance is considerable, since the component Y, for instance, varies from + 15 km./sec. to − 2 km./sec., while z varies from − 200 parsecs to + 200 parsecs.

II. *Interpretation by a Differential Rotation.*

4. Lindblad has suggested the hypothesis that the stars are divided into subsystems, these systems being all the more flat as they revolve with greater velocity. The mean velocities of the stars must then decrease as their distances from the galactic plane increase.

But this is just a hypothesis. Let us, then, simply admit that the mean velocity of rotation varies with the distance from the galactic plane, without specifying whether it increases or decreases.

The numerical results obtained by the authors who have tried to determine the longitude of the centre of galactic rotation can be interpreted in two ways (*M.N.*, March 1930): the longitude of the centre is either 240° or 330°. We are able, with the aid of the preceding hypothesis, to find the longitude of this centre without ambiguity.

Let us draw from a fixed point vectors proportional to the velocity of the sun with regard to groups of stars of various z; the extremities of these vectors will then lie on a straight line parallel to the galactic plane and perpendicular to the direction of the centre of rotation. It is indeed what we have found; according to the figure, the longitude of the centre would be either 320° + 90° = 50°, or 320° − 90° = 230°. Let us select 230°, this being nearly the longitude of the centre for the nearest stars as stated by Seares.

The second asymmetrical drift can thus be accounted for by a differential rotation depending not on the distance of the stars from the centre of rotation, but on their z. This rotation would take place around the centre of the local system (G = 240°).

5. We have tried to specify the longitude of the centre of rotation and it is necessary briefly to describe our method.

Let us consider the stars situated at a distance of r parsecs. Let G_0 be the longitude of the centre of rotation, G and g the galactic longitude and latitude of any direction on the celestial sphere, and V the mean radial velocity of the stars situated in that direction.

The direction cosines α, β, γ, of the direction G, g are:

$$\alpha = \cos G \cos g \qquad \beta = \sin G \cos g \qquad \gamma = \sin g.$$

Let us develop V into a series of spherical functions:

$$V = K + a_1\alpha + a_2\beta + a_3\gamma + 2b_1\beta\gamma + 2b_2\gamma\alpha + 2b_3\alpha\beta + b_4(\alpha^2 - \beta^2) + b_5(2\gamma^2 - \alpha^2 - \beta^2) + \ldots$$

in which $-a_1$, $-a_2$, $-a_3$, are the projections of the velocity of the sun; the terms of the second degree stand for the differential rotation and it is easy to demonstrate that they can be expressed in the form:

$$Ar\cos^2 g \sin 2(G - G_0) + 2Br \sin g \cos g \sin (G - G_0).$$

The first term stands for the differential rotation as regards the distance from the centre (Oort's term), the second one the differential rotation as regards the distance from the galactic plane.

Let us suppose the coefficients b_1 calculated from the data of the observations. We have :

$$\begin{aligned} Ar \sin 2G_0 &= -b_4, \\ Ar \cos 2G_0 &= b_3, \\ Br \sin G_0 &= -b_2, \\ Br \cos G_0 &= b_1. \end{aligned}$$

The first two equations give the value of G_0. Let us call $G_{0,1}$ the longitude thus obtained : then

$$\tan 2G_{0,1} = -\frac{b_4}{b_3}.$$

The last two equations give another value of G_0 which we shall call $G_{0,2}$:

$$\tan G_{0,2} = -\frac{b_2}{b_1}.$$

$G_{0,1}$ is determined within 90° and $G_{0,2}$ within 180°. We have divided the 5500 stars whose radial velocities are known into six groups, according to their distances from the sun. For each of these groups we have developed V up to the terms of the third degree, and we have inferred six values of $G_{0,1}$ and $G_{0,2}$ as in Table II.

Table II.

Distance of the Stars (in parsecs).	Number of Stars.	$G_{0,1}$.		Ar.	$G_{0,2}$.	Br.
		°	°			
50	1129	240·2	or 330·3	+2·14	239·1	+2·05
75	1732	256·0	346·0	+0·36	241·6	+1·71
110	2100	240·0	330·0	+1·31	231·8	+2·70
125	2138	247·0	337·0	+1·57	218·1	+3·01
200	1705	231·0	321·0	+3·28	204·3	+2·98
300	784	246·0	336·0	+4·86	160·6	+3·28

The sign of A in Table II. refers to the solution near 330°.

The value of $G_{0,1}$, the longitude of the centre given by means of b_3 and b_4, is determined only within 90°. It appears in the above table that the values obtained for $G_{0,1}$ are all near 240° or 330°. We have already pointed out that one might hesitate between those two solutions.

The value of $G_{0,2}$, the longitude of the centre given by means of b_1 and b_2, is only determined within 180°. According to the table, it is equal to 240° for the nearest stars, and it progressively decreases down to 160° as the stars become more distant.

6. This can be very easily explained by admitting that the nearest stars do constitute a local cloud which revolves around a centre situated at longitude 240°, while the distant stars revolve around a centre situated at longitude 330°.

When we say " the stars situated at the distance r " we just mean " the stars whose distance is rated r " ; as a consequence of errors of evaluation in the distances, these happen to be dispersed around the mean value r. So it is natural that we should have among the stars " situated at the distance r " a proportion λ of stars belonging to the local system and a proportion $1 - \lambda$ of stars belonging to the galaxy.

When r is small,

$$\lambda = 1.$$

When r is great,

$$\lambda = 0.$$

Starting from such a hypothesis (existence of a local cloud) let us look for its theoretical consequences.

Let $G = 240°$ be the longitude of the centre of the local system ; the longitude of the centre of the galaxy is :

$$330° = G_0 + 90°.$$

Let A and B be the coefficients of the development of V as regards the local cloud system, A′ and B′ the same coefficients for the galaxy.

The development of V for the stars considered will be :

$$\begin{aligned} V = \ldots &+ \lambda[Ar\cos^2 g \sin 2(G - G_0) + 2B\,.\,r\,.\sin g\cos g \sin (G - G_0)] \\ &+ (1 - \lambda)[A'r\cos^2 g \sin 2(G - G_0 - 90°) \\ &\qquad + 2B'\,.\,r\,.\sin g\cos g \sin (G - G_0 - 90°)], \end{aligned}$$

that is :

$$\begin{aligned} V = \ldots &+ [\lambda A - (1 - \lambda)A']r\cos^2 g \sin 2(G - G_0) \\ &+ 2r\sin g\cos g\{[\lambda B\cos G_0 - (1 - \lambda)B'\sin G_0]\sin G \\ &\qquad - [\lambda B\sin G_0 + (1 - \lambda)B'\cos G_0]\cos G\}. \end{aligned}$$

Let us identify this with the development of § 5. We conclude :

$$\tan 2G_{0,1} = \tan 2G_0$$

$$\tan G_{0,2} = \frac{\lambda B\tan G_0 + (1 - \lambda)B'}{\lambda B - (1 - \lambda)B'\tan G_0}.$$

The result is that, if the local cloud does actually exist, $G_{0,1}$ must remain equal to G_0 (or to $G_0 + 90°$), whilst $G_{0,2}$ must vary at the same time with λ, that is at the same time with r.

For the nearest stars $\lambda = 1$, $G_{0,2} = G_0$; as the distances of the stars increase, λ tends towards zero, and $G_{0,2}$ should tend towards $G_0 + 90°$ or $G_0 - 90°$.

Now, this is exactly what we find happening. As the distance from the sun increases $G_{0,1}$ keeps near to both 240° and 330°, whilst $G_{0,2}$ decreases regularly from 240° to 160°. The result of the calculation, $G_{0,2} = 160°$, corresponds satisfactorily to the theoretical result $G_0 + 90°$ or $G_0 - 90°$ since 160° is close to $(240° - 90°)$.

The results strengthen what we have already proposed, namely, that the nearest stars have their centre of rotation at longitude 240°, and the distant stars at longitude 330°.

III. *Second Interpretation.*

7. The existence of the local system is not universally recognised, though the recent researches of Seares confirm the older conclusions of Charlier, Walkey, and Shapley. So it is advisable to ask oneself whether the above results could be differently accounted for.

The longitude of the second asymmetrical drift is about 320°, so it is near the longitude of the vertex of the drifts of Kapteyn ; if one admits that the proportion of stars belonging to one of these drifts depends on z, the mean velocity of the stars must vary with z, this variation having the same direction as the vertex.

However, one does not see clearly the reason of this variation of the proportion in the drifts with z.

Besides it seems that the ellipsoidal hypothesis of Schwarzschild tends more and more to establish itself against the one of Kapteyn, which would ruin this second interpretation.

Lastly, the longitude of the vertex is about 345°, while the longitude of the second asymmetrical drift varies from 330° for the nearest stars to 250° for remote stars.

The study of the B stars that we are about to publish is likely to enable us to settle the possibility of this interpretation, for the B stars do not participate in the drifts of Kapteyn.

IV. *Objections to the Rotation of the Local Cloud.*

8. I am going to enumerate the objections I myself raise against the hypothesis in paragraph 6, and try to answer them.

The first objection is raised by the direction of the rotation. It is well known that the sun is situated on the north of the galactic plane. Let us admit too with Lindblad that the rotatory velocity of the stars decreases as their distances from this plane increase, and let us consider the direction of the second asymmetrical drift.

The conclusion is that the galaxy revolves in the direct way and the local system in the retrograde way.

Now, the dynamical interpretation of Oort's term in the expression for V and of the constant term in the proper motions in longitude has led to the conclusion that the galaxy does revolve in the retrograde direction. Here lies a contradiction with the first interpretation.

9. I do not think this contradiction cannot be overcome : there is a too marked tendency to forget, when considering the dynamical interpretation of the galactic rotation, that one is dealing with the mean velocities of stars.

Such motions are ruled by the equations stated by Jeans, equations similar to those of hydrodynamics.

Let us admit, for the sake of simplicity, that the residual stellar velocities are spherically distributed, and let σ be their dispersion. It is known that $\nu\sigma^2$, where ν is the stellar density, is equivalent to the

pressure p in hydrodynamics. Now, the equations of the motion involve the gradient p,

$$\left(\frac{\partial p}{\partial x}, \quad \frac{\partial p}{\partial y}, \quad \frac{\partial p}{\partial z}\right).$$

It has been supposed zero, but this is a mere hypothesis that ought to be tested by the observations.

So far as I know, only one investigation dealing with the question has been made up to the present.

Strömberg, in 1918, studied the distribution of the residual velocities of the stars as functions of the position of these stars in the sky. The gradient p can be deduced from his investigation by developing the mean residual velocity into a series of spherical functions. However, before calculating the value of p we have thought it preferable to resume these computations with all the material now at our disposal. We shall soon publish the results.

10. Besides, such a research is interesting in another way: the vector gradient p must be situated in the galactic meridian which passes through the centre of rotation, and so we shall have a new means of determining this centre.

Strömberg, in the study I have been alluding to, has thus found for the longitude of the centre three values near 250°. This can be taken as a proof favouring the rotation of the local cloud.

11. The dimensions of the local cloud are very imperfectly known. Kapteyn estimated them at several thousand parsecs, but as Malmquist has shown, his statistical method gives inexact results, because of the very considerable dispersion of the stars' absolute magnitudes. From the distribution of the B stars, Charlier and Shapley evaluate the radius of the local cloud as 800 parsecs and its minor axis 500 parsecs.

These values I consider overestimated, because of the still considerable dispersion of the absolute magnitudes of the B stars.

The results of paragraph 5 would lead one to admit at most 250 parsecs for the radius of the local cloud and 100 parsecs for its minor axis.

12. This point reached, one is tempted to raise against the rotation of the local system the following objection (Eddington). Observation shows that the coefficient A of (Oort term)

$$A \,.\, r \,.\, \sin 2(G - G_0)$$

is about the same for the nearest stars and for the distant stars. If one admits two different rotations for these two kinds of stars, one is obliged to find there an extraordinary coincidence. But this coincidence vanishes if one imagines the local cloud as a kind of vortex in the galaxy, similar to those that occur in a liquid in motion. One would then expect continuity in passing from the local system to the galaxy and that the coefficient A would scarcely vary. The existence of a term K for the nearest stars is accounted for at the same time by the spiral rather than circular deportment of the motion of a vortex. This should equally account for the continuous variation of $G_{0,1}$ from 240° to 150° with the distance from the sun, without using the coefficient λ of paragraph 6.

V. *Does the Asymmetrical Drift of Strömberg depend on Spectral Class?*

13. We have determined the solar motion with respect to the different spectral classes:

Spectral Type.	Apex. (Equatorial Co-ordinates.)		Sun's Velocity.	Apex. (Galactic Co-ordinates.)		Components of the Solar Motion.		
	A.	D.	S.	G.	*g*.	X.	Y.	Z.
*g*M	274·8	34·5	20·9	30·7	19·7	16·90	10·05	7·05
*g*K	276·5	32·1	18·1	27·5	17·4	15·35	7·98	5·42
*g*G	266·8	31·0	18·6	23·7	24·7	15·49	6·80	7·78
F	266·9	28·7	20·4	21·3	24·0	17·38	6·78	8·30
A	264·9	24·0	19·3	16·9	24·2	16·94	4·83	7·93
B	277·8	33·5	22·4	29·5	17·0	16·66	10·83	6·55
*d*G	273·7	25·9	20·9	21·7	17·6	18·49	7·56	6·33
*d*K	279·3	27·2	22·2	24·0	13·2	19·72	8·77	5·07

Fig. 2 represents the projection of the velocity vectors upon the

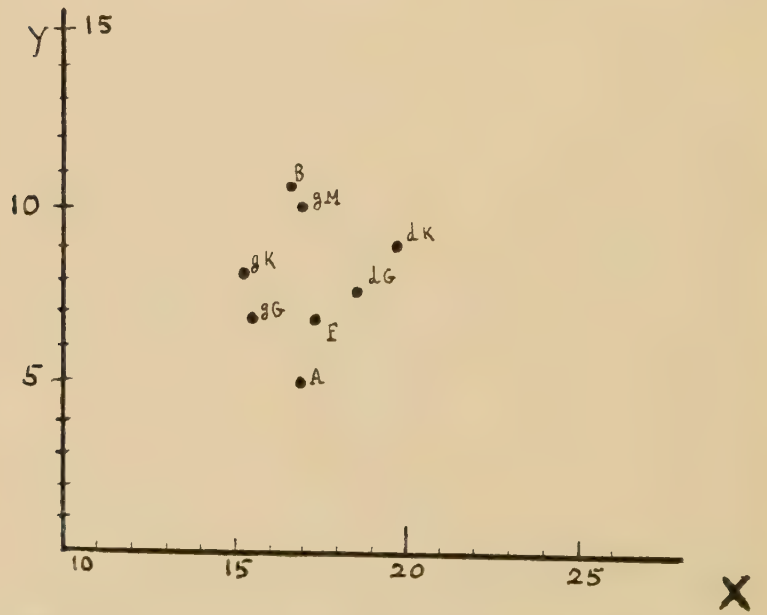


Fig. 2.—The projection on the galactic plane of the velocity vectors of the solar motion with respect to stars of different spectral types.

galactic plane for stars of the different spectral classes. It can be seen that the drift of Strömberg does not appear here. The motion of the sun does not seem to depend on the spectral class.

Summary.

We have brought forward a systematic variation of the velocity of the sun with regard to stars situated at various distances z from the galactic plane. This phenomenon can be accounted for by a rotation of the nearest stars around a centre situated at longitude 240°.

We discuss in detail this interpretation; we think it must not be rejected *a priori*. In any case, the results of the observations showing the variation of the apex, which are described at the beginning of the present study, cannot be doubted, and evidently require an explanation.

I thank M. Jules Baillaud, chief astronomer of the *Carte du Ciel*, who kindly put at my disposal the services of Mlle. Delaveau, and so enabled me to complete the long computations involved in my work.

The Recorded Date of the Chinese Eclipse of − 719.

By J. K. Fotheringham, D.Litt.

I am indebted to Mr. G. L. Wilson of Pasadena for drawing my attention to what he rightly regards as a clerical error in the Chinese date of the earliest of the solar eclipses, cited by Williams from the *Chun Tsew*.* According to Chinese custom the date is expressed by the regnal year of the local prince of Lu, the season of the year, the number of the lunar month, and the name of the day in the cycle of sixty days which goes back to remote antiquity and is still in use. Although we have not sufficient knowledge of Chinese intercalation to identify a Chinese lunar month with precision, the combination of data should normally be sufficient to give an exact date.

In this case Williams gives Kea Tsze as the cyclic name of the day and identifies it with 720 B.C., February 22 of the Julian calendar. Now Kea Tsze in the particular cycle should have fallen on February 17, while February 22 should be Ke Sze, a name very similar in English, but represented by quite different Chinese characters. I find that Legge in his edition of the *Chun Tsew* † has Ke Sze both in the Chinese text and in the translation, and makes no mention of any other reading. I find by computation with the Schram-Schoch tables given in Professor Neugebauer's *Astronomische Chronologie* (1929) ‡ that there was an eclipse of the Sun on February 22 of that year. At Keuh Fow, the capital of Lu, the eclipse should have attained its greatest phase about 20 minutes after sunrise, with a magnitude of 5 digits. There can, therefore, be no doubt that the correct reading is Ke Sze, not Kea Tsze, and that the recorded date agrees with the astronomical computation.

* *M.N.*, **24**, 40, 41 (1863).

† *The Chinese Classics*, v. 1, pp. 10, 12 (1872).

‡ I have applied $+0^{\circ}.1$ to the value of P and have omitted the corrections given by Professor Neugebauer in Table E69, which he has withdrawn as erroneous.

The same eclipse is recorded in the *Annals of the Bamboo Books*. In Legge's edition * the entry runs: "In his 51st year, in the spring, in the 2d month, on the day *yih-sze* (42d of cycle), there was an eclipse of the sun."

This differs from the entry in the *Chun Tsew* in giving the regnal year of the King of China instead of that of the local prince of Lu. There is once more a clerical error in the transliteration of the cyclic name of the day. In the Chinese text Legge gives here, as in the *Chun Tsew*, the characters of the 6th day of cycle, Ke Sze, but he has permitted himself to transliterate the first of these characters as Yih instead of Ke.

It is clear, therefore, that both the *Chun Tsew*, which contains the annals of the State of Lu, and the *Bamboo Books*, which appear to embody the annals of the State of Tsin, date this eclipse − 719 February 22, in agreement with computation.

Students of Williams's article on the eclipses in the *Chun Tsew* should be warned that the position which he assigns to the State of Lu lies far to the south of the position assigned to it on the map by Chalmers, given in Legge's edition. If Chalmers is correct (and I have no doubt that he is) the latitude and longitude of Keuh Fow should be 35°·8 N., 117°·5 E.

University Observatory, Oxford:
1930 *August* 19.

The Cyanogen Band near λ 4200 in the Spectra of Three Cepheid Variables.

By A. Vibert Douglas, M.B.E., Ph.D. (Plates 15, 16.)

(*Communicated by the Secretaries.*)

1. Cyanogen has two well-known bands at λ 4216 and λ 4197. In stellar spectra a wide absorption band extending from about 10 A to as much as 40 A on either side of λ 4200 has been recorded by various writers during the past six or seven years. The attribution of this band to cyanogen molecules in the stellar atmospheres has been made by several astrophysicists at one time or another and appears to be a justifiable assumption at the present time.

The presence of this band in the spectra of A-type stars had been noted by the writer, who tried in 1925, but without success, to make it the basis of a method of estimating absolute magnitudes, since Lindblad had successfully used the cyanogen bands in region λλ3883–3935 for this purpose. Subsequently the band was studied in some F and G stars of the Cepheid variable type, and a cyclical variation in intensity was discovered.

Interest has been stimulated in the band by papers by Shapley †

* *The Chinese Classics*, iii. 1, p. 160 (1865).

† *Harvard Circ.*, 317, May 1928. *Harvard Bul.*, 862, Dec. 1928; 864, Feb. 1929.

and by Elvey.* The latter in his paper refers to what has been previously reported in regard to this band, and goes on to show that as it appears on the spectrograms of the Yerkes Observatory (Bruce spectrograph and 40-inch refractor) the band can be fully accounted for by selective absorption in the glass of the optical system. It is therefore desirable that any data tending to re-establish or confirm the true stellar origin of the band should be recorded.

2. Since all the spectrograms which formed the basis of this study of the behaviour of the λ 4200 band were obtained from the Dominion Observatory, Ottawa, it was essential that the selective absorption, if any, of the optical system of the spectrograph and 15-inch refractor of that institution should be determined.

By the courtesy of the Director and with the kind assistance of Miss M. S. Burland of the Dominion Observatory, the writer was able to test the system in the following manner. A 1000-watt Mazda lamp was attached to the dome and exposures were made of various durations up to five minutes. The plates were developed and subsequently put through the microphotometer in exactly the same manner as for the stellar spectrograms. The resulting microphotometer tracings show no sign of selective absorption in the region about λ 4200, and indeed the transmission throughout the range λ 4100 to λ 4800 is remarkably smooth.

In fig. 1 two tracings are reproduced, one having a linear magnification of × 7, the other × 50, the comparison spectrum in each case being iron and vanadium. The other exposures yielded equally smooth tracings, so that it can confidently be asserted that the absorption in the neighbourhood of λ 4200 on Dominion Observatory spectrograms is not an instrumental effect but an example of true stellar absorption.

3. The assignation of this stellar band to the molecule *CN* is less certain though not improbable. The question of its being due to some molecule other than cyanogen does not appear to require discussion at the present time.† It might be objected, however, that it was in reality merely a spurious band due to the overlapping of the wings of closely adjacent arc or spark lines, or a mixture of the two. This possibility requires careful consideration.

As will be seen in the three sections following, the intensity of this band in the spectra of three Cepheids varies with the phase of the star in each case. This cyclic variation, moreover, is found to be very nearly in phase with the periodic variation in intensity of a large number of typical enhanced lines and of the Balmer lines of hydrogen; and distinctly out of phase with the variations in intensity of the well-known Fraunhofer G band near λ 4300, whose identification with the hydrocarbon molecule *CH* seems to be fairly certain.‡ This would seem to point to the possibility that the λ 4200 band was an ionization effect. A glance, however, at Table I. will show that the central region of the band, λλ 4198–4202, is by no means rich in strong arc lines of stellar

* *Ap. J.*, **70**, 243, 1929.
† *Int. Crit. Tables*, **5**, 412, 1929.
‡ *Ibid.*

importance and completely devoid of intense spark lines. The data here given are taken from Kayser's *Tabelle der Hauptlinien der Linienspektra*.

TABLE I.

Spectrum Lines near λ 4200.

λ in I.A.	Element.	Intensity. Arc.	Intensity. Spark.
4198·313	*Fe* I.	6	3
98·52	*Cr*	3	2
98·72	*Ce*	5	6
98·88	*Ru*	4	3
99·09	*Zr*	6	3
99·10	*Fe*	6	5
99·29	*Y* II.	3	3
99·91	*Ru*	10	10
4201·73	*Ni*	5	..
01·81	*Rb* I.	8	7
02·03	*Fe* I.	7	6
02·44	*V*	1	8

In marked contrast to this portion of the spectrum is the region λλ 4446–4476, where a spurious band is actually produced as a result of the confluence of the wings of more than thirty strong arc lines which are very closely crowded in this region. The variations in intensity of this spurious band, discussed in § 5 below, make it apparent that the λ 4200 band is unlikely to be due to any such cause.

In the absence of any evidence to the contrary, therefore, the band will be regarded as due to stellar cyanogen.

4. *η Aquilæ*.—This star has a period of $7^{d}\cdot176$; a light variation from $3^{m}\cdot7$ to $4^{m}\cdot5$; and a range in spectral class from F2 to G9, the average spectrum being given in the Henry Draper Catalogue as G0p. The variations in intensity with phase, of the Balmer lines and a score of typical enhanced lines as well as of the G band, were determined * recently as far as it is possible to do this from unstandardized plates. The hydrogen and enhanced lines are most intense near the phase of maximum light, whereas the G band is increasing in intensity at minimum light when ionization is less pronounced. This is in full accord with theoretical expectation, conditions favourable to increased ionization being favourable also to dissociation of a molecule, and conversely.

It soon became apparent that the cyanogen band, clearly defined on the majority of the microphotometer tracings of some sixty spectrograms and extending in general from λ 4194 to λ 4212, undergoes a similar variation in intensity, but out of phase relative to the variation of the G band.

* Henroteau and Douglas, *Pub. Dom. Obs.*, ix., 7, p. 163, 1929.

Fig. 2 shows this very clearly. Absolute measures of intensity were obviously impossible, but relative intensities throughout the period were considered to have been obtained by measuring on each tracing the depth of the band below the smooth curve representing the continuous background, and also the depth of an insensitive, closely adjacent line, and then taking the ratio of the two. The G band curve is thus determined by the mean values of the ratio G/λ 4289 plotted against phase, and the *CN* curve likewise for the ratio *CN*/λ 4213. That the resulting curves represent the variations of the numerators rather than the denominators of the ratios is tested by trying out various other lines as the basis of comparison from one tracing to another. Thus in fig. 4 the line λ 4198 is used in addition to λ 4213, and the essential features of

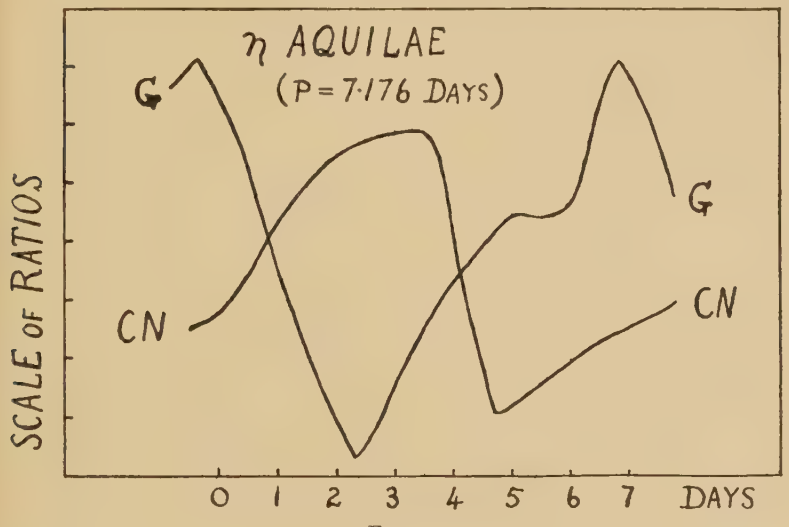


Fig. 2.

the *CN* curve, namely, its phases at maximum and minimum, are practically unaltered. (λ 4289 is an ultimate line of chromium with arc lines of calcium and titanium closely adjacent; λ 4213 is a very weak line of uncertain origin, possibly due to terbium; λ 4198 is an arc line of iron.)

The significance of the curves in fig. 2 can best be appreciated in conjunction with the physical picture of the pulsating star. For brevity the main facts are set out in tabular form (p. 803), the references to radial velocity being taken from a recent paper by Henroteau.*

5. *RT Aurigæ.*—The period of this variable star is $3^{d}\cdot724$; its magnitude changes from $5^{m}\cdot0$ to $5^{m}\cdot9$; its usual spectral classification is F8 with a range from F1 to G5.†

In fig. 3 four curves are given based upon measurements from the

* *Pub. Dom. Obs.*, ix., 5, pp. 136, 142, 1928.
† Cannon, *Harv. Bul.*, 1930 March 1.

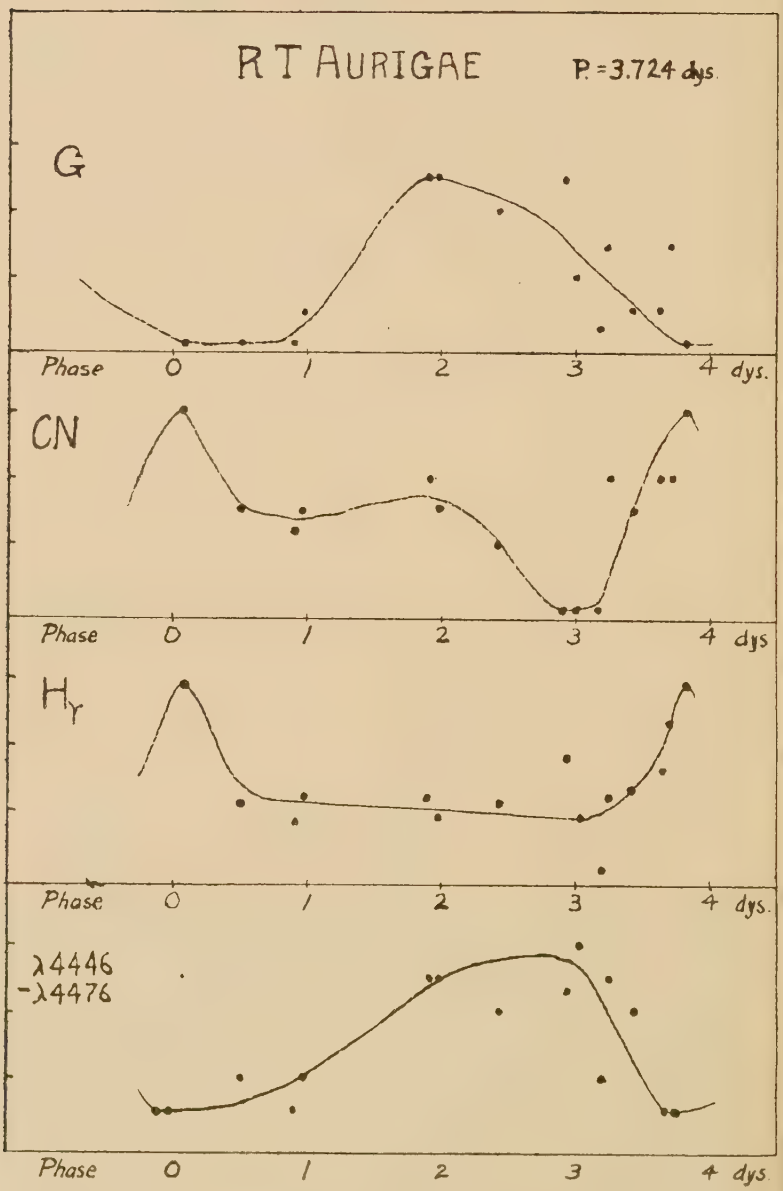


FIG. 3.
(Ordinates represent scales of intensity.)

tracings of fifteen spectrograms. Again the contrast between the G band and the *CN* band is very striking, as also is the similarity between the behaviour of the hydrogen line *Hγ* and *CN*.

TABLE II.

η Aquilæ.

Phase.	Physical State of Star.	Characteristics of Spectrum.
0^d–1^d	Minimum radial velocity; most rapid expansion; maximum light.	Ionization strong or rising; *CN* rising; G falling.
2^d–3^d	Expanding.	*CN* rising; G at minimum.
Approx. $3^d{\cdot}5$	Maximum volume.	Ionization falling; *CN* near maximum.
4^d–5^d	Contracting; minimum light.	Ionization minimum; *CN* minimum; G strong and rising.
Approx. 6^d	Maximum radial velocity; most rapid contraction.	..
Approx. $6^d{\cdot}6$	Minimum volume.	G maximum; *CN* weak but rising.

In order to test how the periodic changes in the intensities of spectral lines would affect a region of the spectrum where the close proximity of strong overlapping lines produces a band-like appearance, measurements were made of the region near λ 4460, where some thirty strong arc lines are closely assembled. This spurious band behaves much as does the G band, as might be anticipated, and in marked contrast to the *CN* band and to enhanced lines.

6. *α Ursæ Minoris.*—This star is recognised as a Cepheid variable having a period of $3^d{\cdot}968$; its range in magnitude is very slight, being $0^m{\cdot}171$ as determined photographically, and less visually. Notwithstanding this small range in light, there are appreciable variations in the spectrum.

Fig. 5 is a reproduction of tracings of four spectrograms representing two distinctly different phases. The G band and the *CN* band are indicated. There is no doubt, as Mr. C. T. Elvey has pointed out, that the better resolution of 3-prism spectrograms greatly reduces the depth of the band, but it does not remove all traces of band absorption, and even if the band does appear somewhat exaggerated on the 1-prism spectrograms, all the measurements being relative, the resulting variations with phase cannot be entirely without value, though lack of standardization of the plates makes certainty obviously impossible.

The G band curve in fig. 4 is based on measurements of ninety-two spectrograms. The *CN* band was measured relative to λ 4198 on seventy-seven spectrograms (*a*), and relative to λ 4213 on thirty-eight (*b*). While the two *CN* curves reflect to some extent the differences in behaviour of these two lines, the main variation represents the changes

in the band and the phases of maximum and minimum are seen to oppose almost exactly those of the G band.

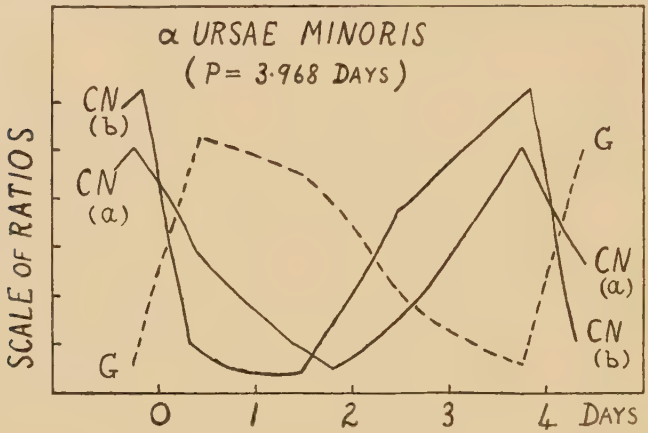


FIG. 4.

(*a*) Relative to λ 4198.
(*b*) Relative to λ 4213.

Considering these curves in conjunction with the curves of radial velocity * and light variation, we may tabulate the phenomena as follows :—

TABLE III.

α Ursæ Minoris.

Phase.	Physical State of Star.	Characteristics of Spectrum.
Approx. $0^d.5$	Minimum radial velocity ; most rapid expansion ; maximum light.	Ionization maximum ; *CN* falling ; G maximum.
$0^d.5$–$1^d.5$	Expanding to maximum volume.	Ionization falling to minimum ; *CN* near minimum ; G strong but falling.
Approx. 2^d	Contracting ; minimum light.	Ionization weak ; *CN* weak but rising ; G strong but falling.
Approx. $3^d.5$	Minimum volume.	Ionization rising ; *CN* rising to maximum ; G falling to minimum.

7. The behaviour of cyanogen as indicated by the foregoing results presents a problem of considerable interest.

The resemblance between its behaviour and that of the enhanced lines suggests at once that the molecule is in the ionized state. Two factors would then be working in opposite directions, the increasing

* *Pub. Dom. Obs.*, ix., 1, p. 52, 1925.

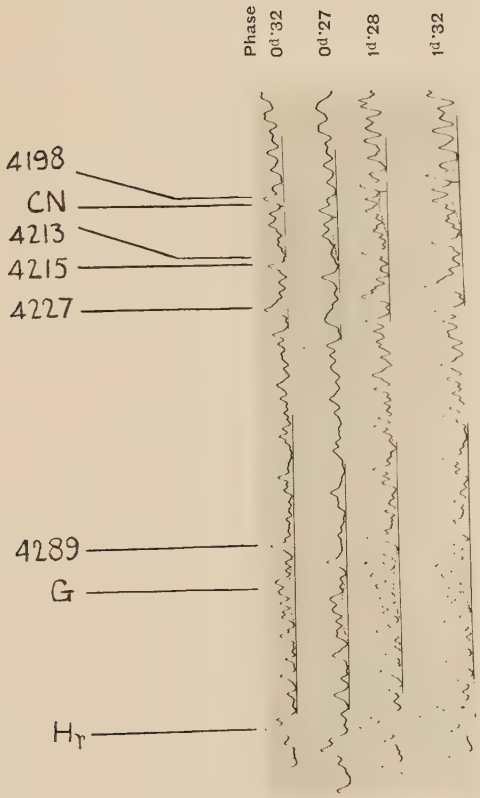


Fig. 5.—Alpha Ursæ Minoris.

temperature bringing about a dissociation of the molecule but producing a larger percentage of ionized molecules among those remaining, while fall in temperature would reverse the two tendencies. No physical data are at present available to disentangle these two effects, but the question is beside the point in any case if the spectroscopists are correct in asserting that the band is undoubtedly due to the neutral molecule.

In the regular sequence of spectral classes the G band reaches its maximum between G and K (according to C. H. Payne), and the *CN* bands near K2 (according to Lindblad). As no one of the above stars falls below class G9, it might be supposed that these two bands would react similarly to the changing physical conditions. The only explanation for the abnormal behaviour of *CN* that suggests itself at the present time is that, like the enhanced lines of iron, titanium, scandium, and so forth, this band has its origin at a very high level in the stellar atmosphere. Why the *CN* molecule should be lifted to a higher level than the *CH* molecule may rest upon the same line of argument as explains the altitude of the ionized calcium atoms in the solar chromosphere, since the *CN* molecule has strong absorption bands (λ 3883, etc.) near the H and K lines of calcium, whereas the *CH* molecule, having no strong absorption in this richest region of the continuous emission spectrum, might not be buoyed up to the high levels to which the *CN* molecule and calcium atoms are carried.

Observations by Shapley and Payne point to the abnormal behaviour of the H and K lines with regard to the periodic changes in spectral class of Cepheids, so that the one explanation may cover the two cases.

Atoms or molecules at high levels in the stellar atmosphere will absorb radiation in accordance with the temperature and pressure existing at that level at any given instant, but there will be a time lag relative to the deeper levels. Increase in interior temperature will precede increase in exterior temperature by a period of time to be measured in days.

This question of time lag plays a fundamental part in the pulsation theory of Cepheid variation, explaining why maximum light coincides with maximum rate of expansion rather than with maximum rate of contraction, this latter being the time when heat is being generated in the interior.

It may well be that consideration of the possibility of different atmospheric levels with consequent time lag provides the key to the problem presented by the apparent anomalies in the behaviour of the cyanogen band and perhaps other features of the spectra of these variable stars.

8. I am indebted to the Director of the Dominion Observatory and to Dr. F. Henroteau for the privilege of examining the spectrograms of that institution, and my thanks are due to them and to all those at the Yerkes, Harvard, and Cambridge Observatories, and at the Imperial College, London, who have at one time or another discussed some aspects of this problem with me.

To Dr. W. M. Smart, Cambridge Observatory, for his generously given assistance and advice, I am especially indebted and very grateful.

Note on the Range in Spectrum Variation of α Ursæ Minoris.
By A. Vibert Douglas, M.B.E., Ph.D. (Plate 17.)

(*Communicated by the Secretaries.*)

1. That α Ursæ Minoris is a variable star having a period of $3^{d}.968$ is well established. The photographic light-curve determined by Hertzsprung * has an amplitude of $0^{m}.171$. A variation in spectral type was recorded by Henroteau † in 1924, but as far as the writer is aware no attempt has been made to determine the range in spectral classification.

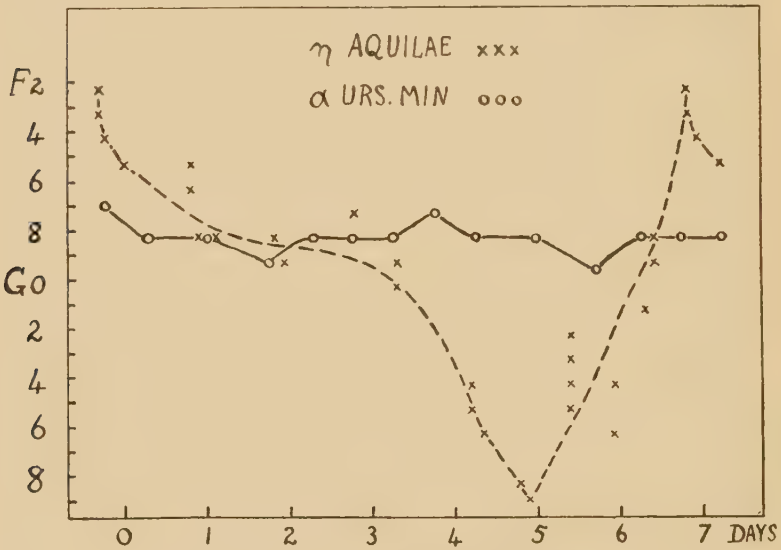


FIG. 1.—Periodic variation of spectral class.

The cyclical changes in the spectrum have been made the subject of an investigation ‡ carried on jointly at the Dominion Observatory and at McGill University, as part of a programme of study of such changes in a large number of Cepheid variables. In this connection microphotometer tracings were made at McGill University of over fifty spectrograms of η Aquilæ and of more than ninety spectrograms of α Ursæ Minoris. These were all 1-prism spectrograms of the Dominion Observatory. These two series, covering every phase of both stars, have formed the basis of comparison by means of which the range of spectrum class of α Ursæ Minoris has been determined.

The period of η Aquilæ is $7^{d}.176$. The manner in which the spectral class changes during this period was noted by Van der Bilt § who gave

* *A.N.*, **189**, 89, 1911.
† *Pub. Dom. Obs.*, ix., 1, p. 52, 1925.
‡ *Pub. Dom. Obs.* (in press).
§ *Ap. J.*, **44**, 286, 1916.

Fig. 2. Microphotometer tracings of spectrograms of (1) η Aquilæ near zero phase; (2) α U Mi near $3^{d}\cdot0$; (3) η Aquilæ near $4^{d}\cdot7$.

as the extreme range A8 to G5. Shapley,* some years later, announced that the range was F2 to G9. In the accompanying diagram, fig. 1, the writer has combined Van der Bilt's observations with Shapley's range by lowering each of the former four spectral class subdivisions, the dotted curve being the result.

In comparing the microphotometer tracings of each plate of α Ursæ Minoris with those of η Aquilæ, four regions of the spectrum were selected for separate comparison, namely :

A—the region about λ 4535,
B— „ „ „ λ 4481,
C— „ „ „ λ 4340,
D— „ „ „ λ 4227.

The spectral class to be associated with each region on each tracing was then determined by means of the broken curve in fig. 1, and recorded, together with the corresponding phase. Averages were found for each group and are shown in Table I., together with the weighted means. In fig. 2 the tracings of three spectrograms are reproduced.

TABLE I.

Range in Spectral Classification of α Ursæ Minoris.

Phase. d d	A.	B.	C.	D.	Mean.
0·0–0·5	F8	F8	F8	F8	F8
0·5–1·5	F8	F8	F7	F7	F8
1·5–2·0	G0	F9	F8	F8	F9
2·0–2·5	F8	F8	F8	F9	F8
2·5–3·0	F8	F8	F7	F7	F8
3·0–3·5	F7	F8	F8	F8	F8
3·5–0·0	F8	F7	F7	F7	F7

Criteria : A—region of spectrum about λ 4535,
B— „ „ „ λ 4481,
C— „ „ „ λ 4340,
D— „ „ „ λ 4227.

2. While it will be seen that the greatest variation given in Table I. is F7 to G0, it should be recorded that individual observations exhibited a much greater range, portions C and D of several spectra of Polaris strongly resembling spectra of η Aquilæ which, from the phase relation of fig. 1, were to be associated with a class as early as F5 or as late as G5. Reference to Van der Bilt's observations, however, shows that in the case of η Aquilæ the estimation of spectral class might vary over four subdivisions for a given phase. To what extent the exceptional extremes in the spectra of Polaris represent real physical extremes in the state of the star or are merely accidental spurious effects due to many causes, such as plate exposure, grain, microphotometer focussing,

* *Harv. Circ.*, **313**, 1927.

sensitivity, etc., it is impossible to say, and little importance is attached to them compared with the mean values.

These means, ranging from F7 to F9, have been plotted against phase, and, as seen in fig. 1, the resulting curve shows two features : (*a*) that the average spectral class is F8, in agreement with the Henry Draper classification ; (*b*) that the variations in the spectrum accompany the variations in magnitude, the maximum F7 being about at the phase of maximum light and the minimum F9 near the time of minimum light.

3. I am very grateful to the Director and to Dr. F. Henroteau for the opportunity afforded me of studying the spectrograms of the Dominion Observatory.

The Effect of Stellar Boundary Conditions : A Reply.

By Sir Arthur S. Eddington.

Although my paper, " The Effect of Boundary Conditions on the Equilibrium of a Star," * was written in connection with a controversy with Prof. Milne, I think that it has some independent value since it gives the formal justification of a rather important step in my treatment of the stellar interior ; I must therefore try to remove any doubt as to its correctness that may have been raised by Milne's rejoinder.† His criticisms fall under four headings : (1) an alleged mistake in differential equation theory ; (2) a " physical difficulty " ; (3) that I have not solved my own problem ; (4) inaccuracy of one of my statements. These are all contained on pp. 682–3 of his paper.

(1) No such mistake was made. Milne's contention is that, when I altered the opacity of the outermost of my three regions from κ to κ_1, I ought consequentially to have changed a coefficient σ in my differential equation to σ_1, since σ is a function of the opacity. But clearly the value of σ to be used in the differential equation is the value pertaining to the region under discussion and not the value pertaining to some other part of the star. I nowhere applied the equation to the outermost region where the change of opacity occurred ; thus no occasion for changing σ arose.

(2) The " physical difficulty " is how (in my problem) the sources contrive to preserve the same relative distribution although slightly altering their strength to conform to the changed surface opacity. But the change referred to in my problem ‡ is a change of *specification* of the star, such as would follow if, for example, we decided to adopt Milne's latest value of the photospheric opacity instead of an earlier one. This would involve correcting our previous calculation of the strength of the sources. This is the change of source-strength that occurs in the problem —a change which can scarcely be pictured as causing physical discomfort to the star.

* *M.N.*, **90**, 279, 1930. † *Ibid.*, p. 678.

‡ The enunciation of the problem is given in the opening paragraph of my paper (*loc. cit.*).

(3) Prof. Milne claims that I have not solved my own problem, because, " after all, L/M is not kept the same." He cannot have read the enunciation of the problem, or he would have seen that there was no requirement that L/M should be unaltered. Indeed, it would make nonsense of the problem to add such a condition, since M is prescribed and L is the quantity to be determined.

(4) The inexactness of a statement may be admitted. I referred to the well-known polytropic solution as the solution corresponding to the case $\kappa_1 = \kappa$. This identification should have been qualified by mention of the end-correction which makes the two solutions diverge from one another close to the actual boundary of the star.

Since Prof. Milne says that my paper replies to an accusation he did not make, I will quote the passage from his paper (which was indicated in my former paper by a reference).

" In any cooling problem boundary conditions are paramount, and they prove to be so in stellar problems. We must not treat the problem as a cooling problem for the innermost $\frac{9}{10}$ of the radius, and then assume the energy flowing past this point can flow through the outer $\frac{1}{10}$ of the radius without affecting the conditions inside."

He adds a reference to an Appendix in which he develops an analogy of a copper sphere and again insists on the necessity of continuing the investigation right up to the boundary in solving the problem of the flow of heat through a star. Presumably, therefore, he wished to stress this contention, and (what particularly concerns me) it is the point which he has taken most pains to impress on the general reader unable or unwilling to read through thirty-six pages of technicalities. Seeing that it is a feature of my investigations that they adopt the procedure condemned by Milne—that is to say, they dispense with any detailed discussion of the outermost $\frac{1}{10}$ of the radius—a reply was called for.

I had not intended to take up any other point raised by Prof. Milne, since it is impracticable to dispose of an unlimited number of points at the rate of two papers per point. But the position is somewhat changed by his recent letter * in which he appears to have found the main origin of the discrepancy between his results and mine. His idea of the constitution of a star has now taken definite shape ; he postulates a small central nucleus of " white-dwarf " density surrounded by a tenuous envelope. This hypothesis, though presenting some difficulties, is not one to be dismissed off-hand ; and it would be premature to judge whether it is likely to have advantages which will render it a formidable competitor to existing views. Meanwhile I may express satisfaction that Milne now recognises that his conflicting results are bound up with a novel suggestion as to the possible constitution of the stars, not contemplated in any existing theoretical discussion, and outside the range of my own investigations which (since 1924) have dealt with perfectly gaseous stars. I am the more surprised that he does not recognise what seems to be the natural corollary—that his earlier suggestions as to the reasons for the divergence, implying faulty working out of the existing theory, were misguided.

* *Observatory*, **53**, 238, 1930.

The Ratio, Sector to Triangle, in Orbit-Determinations adapted to the Arithmometer. By R. T. A. Innes, D.Sc.

Summary.—This paper shows that M. Bazhenov's formulæ deriving the ratio of the sector to the triangle is of more theoretical than practical interest, and that the method of Gauss must still be used to get the best result. The problem is then considered from the machine-computer's point of view and the formulæ modified to suit. A new and short table to derive η from h is given.

The ratio of the sector to the triangle, denoted by η, is important in orbit-work when the first solution is to be corrected, and all the treatises give alternative ways of computing it. The original method due to Gauss may be recalled. Using the suffixes $_1$, $_2$, $_3$ to denote the co-ordinates, etc., attaching to the three dates of observation taken in order of time, and, for the moment, confining attention to the extreme ratio η_2 (sector and triangle formed by the first and third dates), and putting

$$\tau_2{}^2 = f\,(t^3 - t_1)^2$$

(in which the t's are in days and f is the square of the Gaussian constant and is equal to 0·0002959122) and

$$\kappa_2{}^2 = 2(r_1r_3 + x_1x_3 + y_1y_3 + z_1z_3)$$

$$l_2 = \frac{1}{\kappa}\,\frac{r_1 + r_3}{2} - \frac{1}{2}$$

$$m_2 = \tau_2{}^2/\kappa_2{}^3$$

$$\mathrm{W}_2 = (2g - \sin 2g)/\sin^3 g$$

$$w = \sin^2\frac{g}{2}$$

(g = half difference of excentric anomalies)

the Gaussian equations are (omitting the suffixes)

$$\eta^2 = m/(l + w)$$

$$n^3 - \eta^2 = m\mathrm{W}$$

Gauss now puts

$$h = m/(l + \tfrac{5}{6} + \xi)$$

in which

$$\xi = \frac{2}{35}w^2 + \frac{52}{1575}w^2 + \ldots$$

and gets finally

$$\eta^3 - \eta^2 - h\eta - \tfrac{1}{9}h = 0$$

The quantities τ, κ and m are derived from the observations reduced with their respective geocentric distances, but the quantities g, and from it w and then ξ, are not derived directly. However, a recent paper by M. G. M. Bazhenov (*M.N.*, **90**, 324, 1930) shows that the g's can be found directly from the three sets of co-ordinates.

Some of the statements made above require modifications in practice. No direct use is made of the ratios η_1, η_2 and η_3; it is the ratio of the ratios which actually appears in the problem. If we refer to the books (or to Merton's paper, *M.N.*, **85**, 693, 1925) it will be seen that the unknowns in the determination of a first orbit depend upon

$$c_1 = \frac{\tau_1}{\tau_2}\frac{\eta_2}{\eta_1} \qquad \text{and} \qquad c_3 = \frac{\tau_3}{\tau_2}\frac{\eta_2}{\eta_3}$$

and as the τ's (corrected for light-times) are known, the unknowns are essentially the two ratios η_2/η_1 and η_2/η_3. The equations for the three geocentric distances take the form :

$$\rho_1 = \frac{\eta_1}{\eta_1}\mathrm{B}_1' - \frac{\eta_1}{\eta_2}\mathrm{B}_2' + \frac{\eta_1}{\eta_3}\mathrm{B}_3'$$

$$-\rho_2 = \frac{\eta_2}{\eta_1}\mathrm{C}_1' - \frac{\eta_2}{\eta_2}\mathrm{C}_2' + \frac{\eta_2}{\eta_3}\mathrm{C}_3'$$

$$\rho_3 = \frac{\eta_3}{\eta_1}\mathrm{D}_1' - \frac{\eta_3}{\eta_2}\mathrm{D}_2' + \frac{\eta_3}{\eta_3}\mathrm{D}_3'$$

in which the quantities B′, C′ and D′ depend only upon the times and data of the observations (and, as is also implied, the solar places). The ratios η_1/η_2, etc., depend only upon the orbit of the body. The separation of the observational data and the theoretical quantities is thus rendered explicit.

Gauss, and the others after him, only dealt with an individual η, but it is plain that there are relations combining the three. M. Bazhenov indicates these, and it is no longer necessary to find the g's and their dependents w and ξ by approximations.

Putting

$$\left.\begin{aligned} s_2 &= 2r_1r_3 - \kappa_2{}^2/2 \\ \mathrm{S}_2 &= s_1 + s_3 - s_2 \end{aligned}\right\}$$ Similarly for s_1, S_1, etc.

and writing

and

$$\mathrm{T} = s_1\mathrm{S}_1 + s_2\mathrm{S}_2 + s_3\mathrm{S}_3 = \mathrm{S}_1\mathrm{S}_2 + \mathrm{S}_2\mathrm{S}_3 + \mathrm{S}_1\mathrm{S}_3$$

M. Bazhenov gives

$$\tan g_2 = \mathrm{T}^{\frac{1}{2}}/\mathrm{S}_2. \quad \text{(For } g_1 \text{ and } g_3 \text{ change sign.)}$$

With the arithmometer all this work is quickly done. It might now be assumed that, the g's being known, the η's should forthwith be computed by the formula

$$\eta^3 - \eta^2 = m\mathrm{W}$$

or an equivalent one, and M. Bazhenov gives a short specimen of tables by which, with $\tan g$ as argument, η can be found.

Several remarks may be made. The machine-calculator does not like tables of any sort ; $\tan g$ cannot be found without tables ; and the proposed new tables, unless very extended, will demand long interpolations if carried to 7 decimal places ; but the decisive objection is that g is a derived quantity, and in point of fact (as will be shown later) gives the η but weakly. Gauss only makes a third-order use of

g, so that small errors in it have no real effect upon the solution. He uses neither g nor half of its square, but only about a seventeenth of the square of the last quantity—that is to say, the very small quantity ξ, and in many cases, especially if the Hansenian continued fraction is used, ξ may be neglected without appreciable error (see Merton, *M.N.*, **85**, 719, 1925). Nevertheless, when precision is required, there is no doubt that the ξ quantities will be derived more exactly by some modification of M. Bazhenov's process, which uses all three observations, than by Gauss's which uses only two at a time. Whether the refinement is worth the extra work is quite doubtful. The modification hinted at would be to arrange the new process so that w or its square could be derived using the three sets, somewhat as M. Bazhenov has done : if so, then ξ will be found.

Gauss, and others following him, have given an extensive table of $\log \eta^2$ with h as argument, but such a large table is not really required. It is better to compute η by iteration from the formula

$$\eta = 1 + h\frac{(\eta + \frac{1}{9})}{\eta^2}$$

taking the first value of η from Hansen's continued fraction, used as a surd, which is much better for the arithmometer ; thus :

$$\eta = \tfrac{1}{11}[6 + 5\sqrt{(1 + \tfrac{44}{9}h)}]$$

This first value could be taken as the last if the resulting value of η is multiplied by a variable quantity C, which is always near unity, as indicated by Dr. G. Witt in the *V.J.S.*, **65**, 40 (1930) ; but, as printed therein, there is an error, as η computed by the Hansenian surd is always in excess and C must be less than unity, and not greater as given in Dr. Witt's table. In any case, only three approximations to η need to be made, as is shown in Plummer's *Dynamical Astronomy*, § 88, p. 93.

These points may be made clearer by numerical examples, and for this purpose the data of Gauss's classical one of Ceres, 1805–6, are taken over from Oppolzer (French edition, pp. 405–6). Oppolzer gives logarithms, which, in what follows, are always turned into natural numbers.

	1.2	1.3	2.3	
$2rr$	13·198664	13·656328	13·883345	
$\kappa^2/2$	12·236769	9·935534	12·853694	checks, etc.
s	0·961895	3·720794	1·029651	Σ 5·7123391
S	3·6530390	3·7885480	−1·7292479	Σ 5·7123391
sS	3·6441854	−6·4341743	+3·7613553	$\Sigma-$ ·9713664
SS	13·839717	−6·317009	−6·551303	$\Sigma-$ ·971365
		T=−·9713664		
tan g	0·2601469	0·56994679	0·2697971	
g	14° 34′ 55″·56	29° 40′ 51″·06	15° 5′ 55″·45	
	g_1+g_3	29° 40′ 51″·01		
w	0·0161061	·0656014	·0172608	
ξ	150	2557	172	
ξ (Oppolzer)	148	2536	171	

For the reasons given above, it is to be expected that the first set is the better, but it is still more likely that one set is practically as good as the other. Having now the ξ's, we can form the h quantities and get

h	·0454873	·2452556	·0477247

which naturally differ from Oppolzer's values by 2, 21 and 1 units respectively, owing to the differences of the ξ's.

The results of using the g's directly, as indicated by M. Bazhenov, may be given. He puts

$$\eta^3 - \eta^2 = m\mathrm{G} \tan g$$

and, with tan g as argument, has a table for η, which means that from tan g, η is to be found. The best form would seem to be

$$\eta^2 = m \Big/ \left(l + \sin^2 \frac{g}{2}\right) \qquad \sin^2 \frac{g}{2} = w$$

and we have as above

w	·0161061	0656014	·0172608

and from Oppolzer

m	·03878455	·2256279	·04071236
l	·0192977	·0863814	·0197160

giving

η^2	1·0954912	1·48456207	1·10102443
η	1·0466571	1·2184521	1·0492971
Oppolzer	1·0480054	1·2194463	1·0502488

These figures show that the direct use of w influences the 3rd decimal, whereas in the Gaussian method it only influences the 6th or 7th decimal. The η quantities are usually (and satisfactorily) found by approximation. With the first value of η, furnished by the Hansenian approximation, ξ is found from

$$w = \frac{m}{\eta^2} - l$$

and

$$\xi = \tfrac{2}{3\,5} w^2 + \tfrac{5\,2}{1\,5\,7\,5} w^3 + \ldots$$

and this is frequently tabulated: Gauss, table xx.; Watson, table xiv.; Oppolzer, table ix.

Having the h's, we could now compute the η's by any of the well-known processes, much the quickest being by means of the Gaussian table, which furnishes $\log \eta^2$ without undue interpolation. If this table is not available, there are the involved trigonometrical and logarithmic methods of Tietjen and Encke, or there is the approximation given by the Hansenian continued fraction, already referred to. None of these methods is acceptable to the machine-computer. He much prefers to work with no tables or, at worst, with very short tables. This is possible, and one method, as mentioned above, is to reduce the Hansenian continued fraction to a surd and to use it as an approximation, to which the correction is easily tabulated. Here is a specimen of such a table kindly calculated by Dr. C. H. Smiley. The

value of the correction $\delta\eta$ to the surd $[\eta]$ is given in units of the eighth decimal. This table has only 26 entries and η can be found to eight decimals, using a short interpolation to second differences.

h	$\delta\eta$	Δ'	Δ''	h	$\delta\eta$	Δ'	Δ''	h	$\delta\eta$	Δ'	Δ''
·00	0			·09	579		25	·18	3038		22
		1				175				390	
·01	1		8	·10	754		25	·19	3428		21
		9				200				411	
·02	10		11	·11	954		25	·20	3839		21
		20				225				432	
·03	30		17	·12	1179		25	·21	4271		20
		37				250				452	
:04	67		20	·13	1429		25	·22	4723		20
		57				275				472	
·05	124		21	·14	1704		24	·23	5195		19
		78				299				491	
·06	202		23	·15	2003		23	·24	5686		18
		101				322				509	
·07	303		25	·16	2325		23	·25	6195		18
		126				345				527	
·08	429		24	·17	2670		23				
		150				368					
·09	579		25	·18	3038		22				

$$h = m/(l + \tfrac{5}{6} + \xi)$$
$$[\eta] = \tfrac{1}{11}[6 + 5\sqrt{(1 + 44h/9)}]$$

and
$$\eta = [\eta] - \delta\eta$$

In computing the table, η and $[\eta]$ were found to nine decimals. The values in the table are therefore correct to the last place given. Within the limits of the table, and judging by the run of the differences, the correction $\delta\eta$ is always subtractive.

Example (Stracke, p. 30):

$h = 0{\cdot}17399$ $\qquad 44h/9 = {\cdot}85062$

table $0{\cdot}17000$

$$\sqrt{(1 + 44h/9)} = 1{\cdot}360374$$

$n = {\cdot}399$

$$\delta\eta = 2670 + n(368 - 11 + 11n)$$
$$= 2670 + {\cdot}399(368 - 7) = 2814$$
$$\eta = \tfrac{1}{11}(12{\cdot}80187) - {\cdot}00002814 = 1{\cdot}163778$$
(Stracke) $1{\cdot}16378$

As already pointed out, and as indicated by M. Bazhenov's paper, η_2 is not independent of η_1 and η_3. A relation between them can be found as follows: if we write the three fundamental equations from the observations, and put $t_2 - t_1 = \tau_3$, $t_3 - t_2 = \tau_1$ and $t_3 - t_1 = \tau_2$,* we have (only the equations in x are written) †

$$x_1 = f_1 x_2 - g_1 \dot{x}_2$$
$$x_2 = x_2$$
$$x_3 = f_3 x_2 + g_3 \dot{x}_2$$

* The use of the gravitational factor is unnecessary in what follows.

† These f's and g's will not be confused with the earlier ones.

in which

$$g_3 = \frac{\tau_1}{\eta_1} \qquad g_1 = \frac{\tau_3}{\eta_3} \qquad \text{and} \quad g_2 = \frac{\tau_2}{\eta_2}$$

Assuming, as usual in this class of work, that there are no disturbing planets, the f's and g's are closed quantities, and we have rigorously $g_2 = f_1 g_3 + f_3 g_1$, or

$$\frac{\tau_2}{\eta_2} = \frac{f_1 \tau_1}{\eta_1} + \frac{f_3 \tau_3}{\eta_3}$$

In these

$$f_1 = 1 - 2m_1\kappa_1/\eta_1^2 r_2 \qquad f_3 = 1 - 2m_3\kappa_3/\eta_3^2 r_2$$

Using the same Gaussian example with Oppolzer's figures, we have

m	·03878455	·38374016	·04071236	·41284328
κ	4·947075		5·070245	
r_2	2·590181	2·8448353	2·590181	2·8570280
η^2	1·09831525		1·10302256	
f	·86510989	(·13489011)	·85549904	(·14450096)
τ	2·1669655		2·3035974	
η	1·0480054		1·0502488	

$$f_1\tau_1/\eta_1 = 1{\cdot}7887916$$
$$f_3\tau_3/\eta_3 = 1{\cdot}8765365$$

$$3{\cdot}6653281 \quad \text{and} \quad \tau_2 = 4{\cdot}4705539$$

giving

$$\eta_2 = 1{\cdot}2196872$$
$$1{\cdot}2194463 \quad \text{(Oppolzer).}$$

Oppolzer in deriving η_1 only used places 1 and 2, and in deriving η_3 only places 2 and 3, whilst in deriving η_2 he only used places 1 and 3. If the means were taken, then each place would enter equally into the value of η_2.

It should be remarked that in finding the h's and the η's as indicated in this paper, no use whatever is made of trigonometrical or of logarithmic tables, because their introduction would only lengthen the work. With the arithmometer, this paper is complete in itself, once the x_1, x_2, x_3 ; y_1 etc. have been given by the first trials of the geocentric distances. Using the ordinary tables, each extension of the accuracy to be aimed at by using another decimal place entails much extra work ; but with the arithmometer, it is really contracting the number of figures that will give trouble, and it is easiest just to let the machine work to its capacity, noting that in many cases it is only the final result which has to be written down. (In this paper, sets of figures are given which strictly are not necessary.) If two machines were used, it would be little except the end-figures which would have to be written in the course of the work.

The table for h and η extends to $h = ·25$, which is the largest value found used in the books, but if the calculation of η_2 could be made to

depend with precision upon the results for η_1 and η_3, it is obvious that the table carried to h equal to ·10 would be ample.

Addendum.

Incidentally, the following expressions were derived, but they are not essential to the above paper. M. Bazhenov, having tan g, gives two complicated series to compute.

$$W = (2g - \sin 2g)/\sin^3 g$$

The following are simpler. Putting $w = \sin^2\frac{g}{2}$ we have, F(. . .) signifying the hypergeometrical series :

$$W = \frac{1}{3(1-w)}F\left(1, \quad -\tfrac{1}{2}, \quad \tfrac{5}{2}; \quad \frac{-w}{1-w}\right) \quad \{\text{the Hansenian series}\}$$

and

$$= \frac{1}{w}F(1, \quad 2, \quad \tfrac{3}{2}; \quad w) - \frac{1}{w} \quad \{\text{the Gaussian series}\}$$

and there are others.

Johannesburg :
1930 *April* 21.

A Screw-and-nut Drive for Equatorials. By F. J. Hargreaves.

The following is a brief account of an unusual equatorial drive which has just been completed with satisfactory results and which seems to possess certain advantages over the usual form of drive.

In the ordinary worm and worm-wheel drive it is not only defects in the worm or screw that are responsible for inaccurate following ; the teeth of the worm-wheel or driving-circle also play a part. It is very difficult to distinguish errors due to the teeth from those due to the screw, and those due to the teeth are difficult to avoid or correct. Moreover, the arrangement presents very little opportunity for introducing automatic compensating devices. For these reasons really good drives of standard form are costly ; and even the best of them, unless of very large radius, may have small ineradicable residual errors.

In the new arrangement these disadvantages are avoided ; there are no worm teeth, practically the only source of error (apart from the driving clock) is the screw itself, and there is an opportunity of automatically correcting any errors which may be present, whether due to the screw or to the gearing between the clock and the screw.

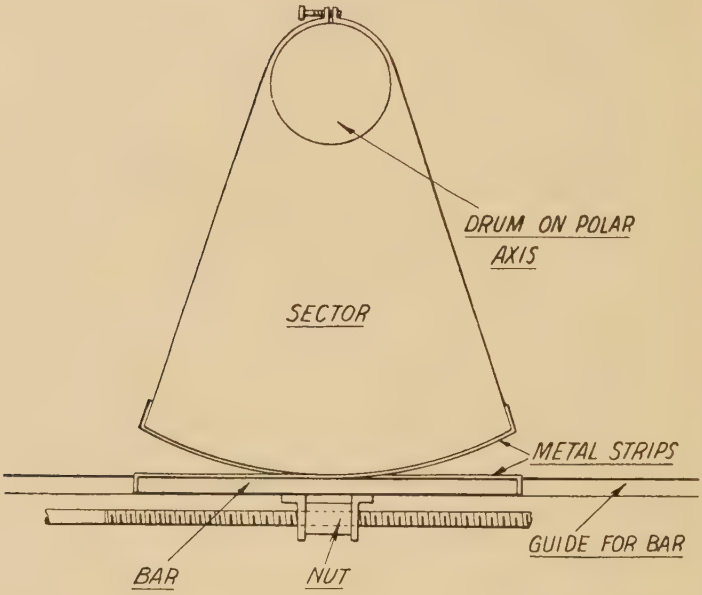
The new drive consists of a long screw engaging a nut carried on a straight bar sliding in guides, the motion of the bar being transmitted positively to a smooth-edged sector by means of opposed flexible steel

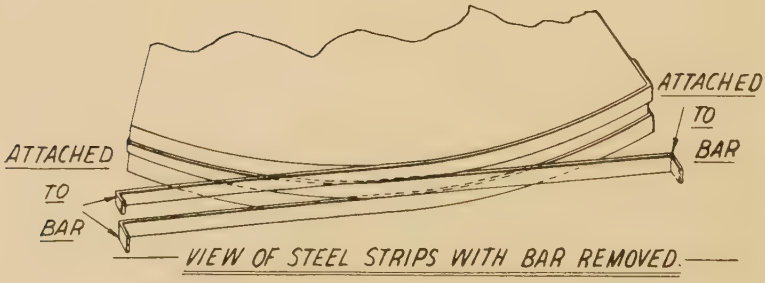
strips, in the manner of certain gear-cutting machines.* The diagrams show the arrangement clearly, the second view in particular showing how the steel strips anchor the sector to the bar while allowing rolling of the one on the other.

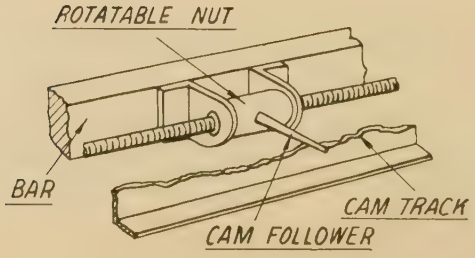
An experimental arrangement on these lines has now been thoroughly tested. The screw (a lathe lead-screw) cost only a few shillings and obviously had no pretensions to accuracy, but with a sector of 23 inches radius the periodic error nowhere exceeded 7″ total amplitude, and in the mean was only about 4″. After the thread had been lightly "lapped" and the abutment shoulder trued up, the error was greatly reduced, being for the most part almost inappreciable and rising to a maximum of about 3″ total amplitude in one or two places. It is recognised that the thread of the nut also plays a part in these errors, but as it is practically impossible to ensure that the axis of the nut shall be exactly coincident with the axis of the screw, it may be taken for granted that the thread of the nut allows most of the errors of the screw to manifest themselves. Errors of period shorter than the period of rotation of the screw will, however, be much reduced by the use of a nut (owing to the fact that it engages the whole circumference of the screw), and for this reason alone the nut has a great advantage over a worm-wheel. An attempt to eliminate the one-rotation period by mounting a very short nut in a gimbal ring failed completely, probably owing to excessive friction at the gimbal pivots.

It is evident that if the nut is mounted to rotate in bearings about the axis of the screw, the motion transmitted to the sector by the screw can be varied by rotating the nut itself. The intention is to provide the nut with a projecting stylus moving over a waxed surface and to guide on a star by rotating the nut while the screw is rotated uniformly by the driving clock, which is pendulum-controlled. The stylus will thus trace in the wax a path representing all the errors of the drive and including, of course, the R.A. component of refraction. A cam track cut to the shape thus traced out will then be placed in the position previously occupied by the waxed surface, and the stylus will be replaced by a cam-follower bearing on the track, thus causing the nut to rotate, as it traverses the screw, always in exactly the same way as it does if rotated by hand to follow the star. Consequently, assuming absence of slackness in the bearings of the nut and freedom from dust, the errors will be automatically corrected with a degree of accuracy limited only by the errors of the initial guiding. The arrangement is shown in the lowermost view, in which, of course, the waviness of the cam track is much exaggerated. The effect of refraction could be eliminated, if desired, either by altering the curve to the extent necessary after it has been traced, by applying a correction at intervals

* After this drive had been designed, the writer's attention was directed to the fact, previously unknown to him, that a flexible band drive had previously been used for equatorials—for example, in some early instruments made by the Clarks. In these instruments, however, the worm and worm-wheel were retained and the flexible bands merely used to transmit the rotation from the worm-wheel to the polar axis.


DRUM ON POLAR AXIS
SECTOR
METAL STRIPS
GUIDE FOR BAR
BAR
NUT


ATTACHED TO BAR
ATTACHED TO BAR
VIEW OF STEEL STRIPS WITH BAR REMOVED.


ROTATABLE NUT
BAR
CAM TRACK
CAM FOLLOWER

during the guiding, or by tracing the curve in a succession of short lengths, each length on a star crossing the meridian at a considerable zenith distance.

A similar cam-track arrangement is used in precision screw-cutting lathes, in which the shape of the track is determined by measurements of the errors of the screw. It is tempting to imagine a drive of the kind herein described being used to determine the errors of lathe lead-screws and to trace the cam tracks to be used with them.

The guiding motion produced by rotating the nut is of great delicacy, and the correcting track would seem to afford a means of obtaining accuracy of a very high order when the apparatus is made to professional standards and the whole length of the screw is completely enclosed to exclude dust and grit.

From a different point of view—that of the amateur constructor—the arrangement offers advantages of another kind in that it can be made without machine tools and elaborate equipment, and the lead-screws and nuts, being commercial productions, can be obtained at a very low price. Of course most amateurs would not find it necessary to provide correcting gear, but would obtain a reasonable degree of accuracy by using sectors of long radius. The " English " mounting is particularly suitable, there being usually plenty of room for a large sector in this form of mounting.

It is improbable that the correcting gear will be constructed in the present instance. The variations of pitch along the length of the screw are very small, and the short-period errors are diminishing surprisingly quickly merely as a result of repeatedly traversing the nut along the screw by hand in the course of testing. For these reasons it seems likely that within a short time the drive will be sufficiently accurate for all the needs of an amateur astronomer, without any correcting gear.

Summary.

The paper briefly describes an equatorial driving arrangement consisting of a long screw (actually a commercial lathe lead-screw) engaging a nut fixed to a bar guided in a straight path and transmitting its motion positively to a plain sector by means of opposed flexible steel strips. There is also described a projected automatic correcting device involving rotation of the nut itself, such rotation being caused by a cam track, the shape of which is traced out by a preliminary traverse during which the nut is rotated manually to guide the telescope on a star.

An Active Region of the Sun on 1930 *August* 12.

By H. W. Newton.

(*Communicated by the Astronomer Royal.*)

(1) The observations described in this note were made at the Royal Observatory, Greenwich, with a spectrohelioscope designed by Dr. Hale and lent by the Mount Wilson Observatory. The spectrohelioscope has been fully described by Dr. Hale in the *Astrophysical Journal*, **70**, 265–311, 1929. It suffices to add that the instrument at Greenwich is fitted with rotating square prisms, according to Dr. J. A. Anderson's design, and that the spectrum is formed by a Rowland grating ruled with 14,438 lines to the inch. The observations were made in the light of $H\alpha$ in the first order spectrum, of which the scale is approximately 1 mm. = 4A. The width of the second slit, isolating a portion of the $H\alpha$ line, was usually 0·1 mm. The diameter of the monochromatic image of the sun's disc formed by the optical train at the second slit is 2 inches, of which a strip ·25 inch wide and 1·1 inch long is rendered visible by the rotating prisms. The eyepiece used magnifies twice.

(2) The observations relate to a region of the sun containing a moderately large sunspot that crossed the central meridian on August 12 at 6^h G.C.T. (longitude of spot 73°·9; latitude 6°·8 S.). The spot was one of frequent type, being roughly circular and fairly constant in area and position. It was either a return or a new formation in the place of a circular spot which began on July 11 near the sun's east limb in the previous rotation. In both rotations several dark filaments possessing considerable radial motions were observed on various occasions in the immediate vicinity of the spot, but so far as the Greenwich observations are concerned the region became suddenly and unusually active about 11^h on August 12. The phenomena observed naturally fall under two headings :—

(*a*) Intensely bright flocculi with little or no radial velocity.

(*b*) Dark markings with large radial velocities.

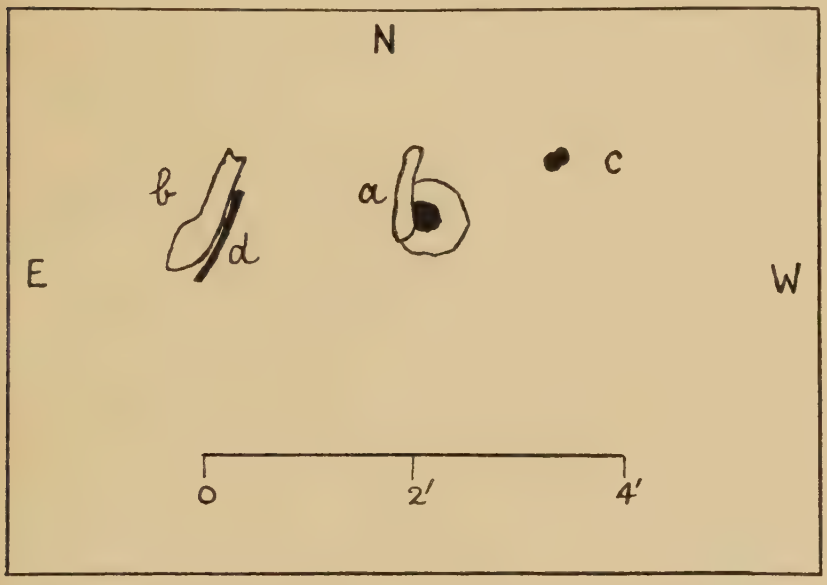
(*a*) *Bright Flocculi.*—On August 12 the region of the sunspot then on the central meridian was observed about 9^h G.C.T. Cloud then intervened, and it was not seen again until 11^h. In this interval of two hours a striking transformation had taken place. The bright hydrogen flocculi, which were not unusual in appearance at the time of the first observation, had become brilliant in two patches, one, *a*, immediately north-following the umbra of the spot and partly overhanging it; the other, *b*, about 2′ following the spot (see diagram). These brilliant areas were kept under observation * until just after 12^h, when the sky, already cloudy, became overcast. During the hour's observations appreciable changes in shape of the bright patches were noted. Two measures, made 15 minutes apart with the "line-shifter," of the wave-length at which the flocculi appeared most intense gave a

* Mr. Greaves observed them for a short while about $11^h\ 55^m$.

very small shift towards the violet, which interpreted as being due to line-of-sight motion was equivalent to − 2 km./sec.*

When the sun was next observed at about 14^h, the formation of the bright flocculi had entirely changed and diminished in brightness. A few very bright scattered points were left, and there was a narrow bright bridge that crossed the umbra of the spot. The flocculi were brightest at the setting of the line-shifter, indicating a radial velocity of − 6 km./sec.* The following day at 10^h the bright flocculi were not conspicuous.

The brightness of the flocculi on August 12 was so striking that it seemed possible that they might be visible in light of other wave-



lengths. Between $11^h\ 10^m$ and $11^h\ 40^m$ a few observations were made (with second slit ·06 mm. wide) in the line D_2, the region of D_3, in b_1 and $H\beta$. In the magnesium line, b_1, a structure like branching faculæ could be faintly seen, but, except perhaps for a marking which seemed to touch the penumbra of the spot on its following edge, the structure did not coincide with the brilliant patches seen in *Hα*.

(*b*) *Dark Flocculi.*—Contemporaneously with the observations made of the brilliant *Hα* flocculi, two dark flocculi made their appearance in the same region. At $10^h\ 59^m$ a short curved dark marking (*c* in diagram) was seen north-preceding the spot, and its radial velocity measured as −33 km./sec. This measure was obtained in the manner

* Eighty-six measures of bright flocculi near the centre of the sun's disc between 1930 March 8 and September 4 give a mean displacement equivalent to $-2{\cdot}8 \pm 0{\cdot}3$ km./sec.

explained by Dr. Hale by turning the line-shifter (thereby gradually changing the wave-length of the light passing the second slit) until the object was seen at its darkest. At $11^h\ 8^m$ this small marking had become nearly circular and almost as dark as the umbra of the spot. Its measured radial velocity was again − 33. At $11^h\ 19^m$ the marking could not be seen.

The second dark marking (*d* in diagram) was suspected at $11^h\ 45^m$ on the south-preceding edge of the other and larger mass of bright flocculi, *b*. At $11^h\ 50^m$ it was unmistakable as a short curved flocculus with a measured velocity of + 47 km./sec., and at $11^h\ 58^m$ differential velocities were clearly indicated along its length ; that is to say, that as the line-shifter was slowly turned, successive portions of the flocculus became visible from one end of it to the other. The measures gave for the north end + 33 km./sec. and for the south end + 116 km./sec. At 12^h the measures were repeated (unavoidably while thin cloud was passing) and gave + 26 and + 89 km./sec. Further observations were then impossible, and on resuming them at 14^h the marking had disappeared.

(3) It is perhaps of interest to give a short summary of measures of dark flocculi with large radial velocities seen in this region before and after August 12. The velocity given in each case is the measured component of the radial velocity in km./sec.

July 14.—9^h. Leader spot 22° east of central meridian. Between the leading and following sunspots, a short dark flocculus with progressive velocities − 19 to + 38.

August 7.—12^h. Spot 63° east of C.M. Small circular marking south of spot ; velocity + 29. Crossing the bright flocculi following the spot was a short dark marking with progressive velocities along its length from − 35 to − 5.

August 8.—9^h. Spot 51° east of C.M. Following the spot a curved dark marking on the preceding edge of a bright flocculus that terminated in a brilliant spot at its north end. Progressive velocities along the length of the dark flocculus from − 23 (north end) to + 34 (south end).

August 14.—9^h to 10^h. Spot about 30° west of C.M. A circular black marking, about 2′ north-preceding, comparable in appearance with the umbra of the sunspot; velocity at $9^h\ 25^m$, − 40, at $9^h\ 35^m$, − 36. At $9^h\ 35^m$ a companion dark marking appeared with measured radial velocity + 29. At $9^h\ 43^m$ a short curved filament was seen south-preceding the sunspot with velocity − 32. At $10^h\ 0^m$ another curved filament appeared, ending in a black head at the north-following edge of the penumbra of the sunspot. Measured radial velocities ranged from + 6, for the end away from the sunspot, to + 35 at the black head itself, which was alone visible at this setting of the line-shifter.

August 16.—9^h to $10\frac{1}{2}^h$. Spot about 55° west of C.M. Between $9^h\ 10^m$ and $9^h\ 54^m$ an intense black mark with a small companion preceding had formed about 2′ north of the sunspot. A number of measures of the component of radial motion were obtained and are tabulated

below as an illustration of the rapidity of the changes taking place, which were also apparent in the change of shape of the marking.

TABLE I.

G.C.T. 1930 Aug. 16.	Radial Velocity in km./sec.	Remarks.
h m		
9 10	marking not visible	..
9 54	−46	About $\frac{3}{4}'$ long, very black.
9 59	−31	1′ long, not quite so dense.
10 4	−24	Has broken into two parts.
10 10	−20	..
10 17	−10	A dot.
10 27	−10	A dot, barely visible.
10 48	−14	Black dot.

(4) *Accuracy of Measures.*—It is hoped to discuss the accuracy of the measures of radial velocity made with the spectrohelioscope in a future paper, presenting the results of observations made of a number of bright and dark flocculi. The probable errors of the above measures are believed to be of the order of 2 km./sec. Zeros of the *Hα* line, referred to an undisturbed portion of the sun's disc near the centre, were taken frequently during the observations.

(5) The ephemeral but intense brightening of *Hα* flocculi in the region of sunspots has been registered in the past on spectroheliograms on five occasions—1908 September 10, 1926 February 22 and October 13, 1928 January 26 and September 14. Between 1926 January 24 and 27 four distinct occurrences in the same region were seen visually by Hale observing with a spectrohelioscope. On 1926 February 22 spectroheliograms were secured at Kodaikanal in K_{23} as well as in *Hα*, and they showed that concurrent activity was taking place in the associated calcium flocculi. It seems, therefore, that intensely bright calcium flocculi previously registered on H or K spectroheliograms on four occasions represent the same phenomenon.

A list of sudden and intense local brightenings of the sun's surface is given in Table II.,* to which the historic observation of 1859 September 1 may be added. The first column gives the date and estimated time (G.C.T.) of maximum brightness. The second column gives the Greenwich number of the associated group of sunspots; the third column its solar latitude. The fourth column gives the distance in longitude east or west from the sun's central meridian at the time of maximum brightness. The fifth column gives the G.C.T. of commencement of any magnetic storm recorded at Greenwich that closely followed the time of the solar disturbance. The lower limits of ranges in D, H, and V of the magnetic storms were taken as D = 30′, and H or V = 150γ.

A few comments may be made concerning Table II.

* In compiling this table I am indebted to Mr. Evershed for assistance and to Dr. Hale's papers.

TABLE II.

Date and Time, G.C.T.	No. of Sunspot.	Latitude.	Distance from C.M.	Magnetic Storm.	Reference.
1859 Sept. 1·5	520*	20° N.	12° W.	Sept. 2·2	Carrington and Hodgson, *M.N.*, **20**, 13, 1859.
1892 July 15·7	2583	31° S.	22° W.	July 16·5	Hale, *Astronomy and Astrophysics*, **11**, 611, 1892.
1908 Sept. 10·4	6519	6° S.	18° W.	Sept. 11·9	Fox and G. Abetti, *Ap. J.*, **29**, 40, 1909.
1909 Sept. 24·4	6728	5° S.	8° W.	Sept. 25·5	A. Fowler, *Nature*, **81**, 396, 1909. W. J. S. Lockyer, *M.N.*, **70**, 12, 1909. Slocum, *Ap. J.*, **31**, 27, 1910.
1909 Sept. 28·2	6728	5° S.	60° W.	None.	Evershed and Sitarama Aiyar, *Kodaikanal Bull.*, **22**, 1910.
1916 June 29·2	7751, 54, 56	12° N.	16° W.	None.	Evershed, *Observatory*, **43**, 411, 1920.
1926 Jan. 25·8†	9861	21° N.	17° W.	Jan. 26·7	Hale, *Ap. J.*, **70**, 309, 1929.
1926 Feb. 22·2	9881	23° N.	10° W.	Feb. 23·7	Royds, *M.N.*, **86**, 380, 1926.
1926 Oct. 13·6	10065	17° N.	3° E.	Oct. 13·8	D'Azambuja and Grenat, *L'Astronomie*, **40**, 489, 1926.
1928 Jan. 26·8	10504	21° S.	44° E.	None.‡	Richardson, *P.A.S.P.*, **40**, 126, 1928.
1928 Sept. 14·3	10733	15° N.	23° W.	Sept. 18·7§	Evershed. Unpublished data.
1930 Aug. 12·5	not numbered	7° S.	3° W.	Aug. 14·8	Newton. Present data.

(i) In each of the twelve cases, the brightening took place in the immediate vicinity of a sunspot; in about half the cases it was chiefly over the spot itself.

(ii) With one or two exceptions, the associated spot was to a lesser or greater degree unusual in character. Areas of spots in Table II. range from 350 to 3285 millionths of the sun's hemisphere.

(iii) The brightening was of short duration, ranging, in the best recorded cases, from about $\frac{1}{4}$ hour (5 minutes in the case of the 1859 September 1 observation, which being made in integrated light is not, however, strictly comparable) to 7 or 8 hours.

(iv) In connection with paragraph 2 (*a*) and (*b*), pp. 820–1, it is of interest to note that—

(*a*) On 1926 January 25, the lines D_1, D_2, and D_3 were brightly reversed in the spectrum of the large spot, and on 1926 February 22 reversals of D_1, D_2, and of the *b* lines were observed in a similar position. On both occasions D_3 appeared dark away from the spot.

(*b*) On 1908 September 10, 1909 September 28, 1926 January 25, 1926 February 22, and 1926 October 13, as well as in the

* Carrington's enumeration.

† Four separate eruptions were seen between January 24 and 27.

‡ A magnetic disturbance, with range in H just below that of a small storm, commenced on January 27·1.

§ It seems doubtful whether this small storm, following $4\frac{1}{2}$ days after the solar disturbance, should be included in the table.

present instance, there was evidence of concomitant dark flocculi with large radial velocities of the order of 100 to 150 km./sec. For instance, on 1908 September 10, visual and photographic examination of the spectrum showed that there was, first, a violent uprush of hydrogen immediately behind the spot, and 2 hours later a descent in two places with velocities of 100 and 170 km./sec. respectively.

(v) On seven out of twelve occasions in the above table a magnetic storm commenced within $1\frac{1}{2}$ days after the solar disturbance. Each of these seven storms was a "great" storm, *i.e.* the range in D = 60′, or in H or V = 300γ. In the present instance (1930 August 12) a small magnetic storm commenced two days later. The magnetic traces were, however, disturbed almost continuously from August 6 to August 15, so that the small storm of August 12 to 13 may be a chance coincidence.

In two of the three cases, where no magnetic storm followed, the solar disturbance took place when the region was directed appreciably away from the earth.

(6) *Summary.*—The chief facts relating to the phenomena of 1930 August 12 seem to be as follows :—

(*a*) The sudden appearance of brilliant hydrogen (*Hα*) flocculi in the region of a sunspot that was accompanied by the usual bright flocculi.

These brilliant areas of rapid rise and decline in luminosity showed no appreciable radial motion in excess of a small outward velocity found on the average for bright *Hα* flocculi.

(*b*) The contemporary appearance of dark hydrogen flocculi possessing large radial motions. The first of these flocculi, an almost circular marking of short duration, had a measured velocity outwards of about 30 km./sec. Later, a curved dark flocculus bordering one of the brilliant areas showed accelerating velocities of descent from about 30 to 100 km./sec.

(*c*) The associated sunspot was in appearance one of ordinary type, and there were no obvious changes in it after the advent of the activity seen in *Hα* light ; but

(*d*) The region was active both before and after August 12, as shown by the appearance on several occasions of dark flocculi moving inwards or outwards with measured radial components of 30 or 40 km./sec.

(*e*) Although a small magnetic storm took place on August 14 to 15 it cannot with certainty be associated with the solar disturbance of August 12, the magnetic traces being disturbed almost continuously from August 6 to August 15.

Dr. Hale has drawn attention to the need of watching for these solar disturbances seen in *Hα* light that give, as he points out, a considerable correlation with terrestrial magnetic storms. It seems likely that the general use of spectrohelioscopes will show that such solar disturbances, by nature short-lived, are more frequent than would appear from past records.

Royal Observatory, Greenwich,
1930 *September* 5.

Observations de la comète Beyer (1930 *b*) *faites à l'equatorial Doridis* (0m·40) *de l'Observatoire National d'Athènes.* Par G. Adamopoulos.

(*Communicated by Dr. L. J. Comrie.*)

Date, 1930	T.U.	Δα	Δδ	Nombre de Comparaisons.	α	l.f.p.	δ	l.f.p.	*
	h m s	m s	′ ″		h m s		° ′ ″		
Mars 29	19 54 55	−1 8·74	−5 22·95	10 : 10	6 9 41·37	9·699	+39 26 26·15	0·142	1
Avril 3	20 16 31	−1 29·73	−1 23·91	10 : 10	6 13 8·16	9·749	+41 5 58·59	0·343	2
12	20 5 17	+0 12·62	−8 18·97	12 : 10	6 21 35·50	9·806	+43 48 30·36	0·539	3
15	19 33 36	−0 7·11	−5 34·32	12 : 9	6 24 59·52	9·767	+44 38 23·17	0·299	4
16	19 25 45	+0 45·98	−4 9·86	10 : 10	6 26 11·39	9·765	+44 53 27·64	0·197	5
24	19 33 41	−0 46·86	+0 41·19	9 : 11	6 37 6·91	9·805	+46 58 51·87	0·287	6
25	19 21 40	−1 34·35	+4 7·09	11 : 11	6 38 36·73	9·799	+47 12 29·33	0·234	7
26	19 52 32	−0 50·29	−5 14·72	10 : 10	6 40 11·21	9·822	+47 27 11·30	0·379	8
28	19 39 56	+1 32·49	+9 45·92	10 : 10	6 43 20·75	9·822	+47 55 13·43	0·338	9
Mai 1	20 48 33	−1 40·14	−2 5·91	10 : 10	6 48 26·83	9·845	+48 37 4·73	0·519	10
5	20 40 48	+0 22·64	+0 23·06	10 : 10	6 55 34·21	9·853	+49 29 28·74	0·508	11
8	20 1 20	+2 20·16	+3 27·19	10 : 10	7 1 11·25	9·857	+50 6 42·92	0·471	12
19	21 32 45	−0 59·86	+4 6·79	10 : 10	7 24 30·69	9·844	+52 12 54·20	0·733	13
31	19 59 36	+0 14·82	−9 5·85	10 : 10	7 53 53·20	9·897	+54 6 4·03	0·544	14

Positions moyennes des étoiles de comparaison pour 1930·0.

*	Gr.	α	δ	Autorités
		h m s	° ′ ″	
1	8·5	6 10 50·11	39 31 49·10	A.G. Lund No. 3183
2	7·7	6 14 37·89	41 7 22·70	„ Bonn 5162
3	8·9	6 21 22·88	43 56 49·33	„ „ 5243
4	9·2	6 25 6·33	44 43 57·49	B.D.+44° 1460
5	9·1	6 25 25·41	44 57 37·50	B.D.+45° 1309
6	9·1	6 37 53·77	46 58 10·68	B.D.+47° 1331
7	8·1	6 40 11·08	47 8 22·24	A.G. Bonn 5471
8	8·4	6 41 1·50	47 21 56·58	„ „ 5481
9	9·3	6 41 48·26	47 45 27·51	B.D.+47° 1341
10	8·2	6 50 6·97	48 39 10·64	A.G. Bonn 5594
11	9·3	6 55 11·57	49 29 5·68	„ „ 5655
12	9·0	6 58 51·09	50 3 15·73	„ „ 5709
13	7·2	7 25 30·55	52 8 47·41	„ Cambridge, Mass. 2866
14	9·2	7 53 38·38	54 15 9·88	„ „ „ 2996

Remarques.

29 Mars.—La comète a l'aspect d'une nébulosité à bords mal définis. Elle présente une condensation intense, qui donne l'impression d'un noyau de 12^{e} grandeur environ.

12 Avril.—La comète paraît très faible à cause de la Lune (13 jours). Vers la fin de l'observation elle est à peine visible.

15 Avril.—Une étoile de 9·5 gr. très près du noyau de la comète gêne l'observation.

1 Mai.—Nébulosité presque ronde d'un diamètre de 1′ 30″ environ ; noyau central de 13^{e} grandeur à peu près.

19 et 31 Mai.—La comète paraît très faible à cause de la petitesse de son altitude au dessus de l'horizon. Le noyau étant presque invisible, les pointés ont été effectuées sur le centre de la nébulosité.

LIST OF ADDITIONS

TO THE LIBRARY

OF THE SOCIETY

1929 JUNE 30 TO 1930 JUNE 30.

Serial Publications received by exchange in the regular course, and Reprints of Papers published in some of these Serials, are not included in this Annual List.

An asterisk (*) indicates that the work is an excerpt.

Abbadia, Observatoire:

——: Catalogue de 14263 étoiles comprises entre +16° et +24° (Zone photographique de Paris), observées en 1899, 1900, 1901, 1902, 1903, 1904, 1905, 1906. Réduites à 1900·0.
(*Observatory.*) 4to. Hendaye, 1915

——: Catalogue de 1343 étoiles, dont 720 fondamentales, comprises entre +43° et −26°, observées en 1919, 1920, 1921. Réduites à 1920·0.
(*Observatory.*) 4to. Hendaye, 1923

——: Deuxième Catalogue comprenant 1332 étoiles, dont 711 fondamentales, de +45° à −26°, observées en 1922, 1923. Réduites à 1920·0.
(*Observatory.*) 4to. Hendaye, 1928

——: Troisième Catalogue comprenant 1250 étoiles, dont 658 fondamentales, de +45° à −26°, observées en 1924, 1925. Réduites à 1925·0.
(*Observatory.*) 4to. Hendaye, 1930

——: Observations faites au Cercle Méridien en 1920 et 1921. Tome 14.
(*Observatory.*) 4to. Hendaye, 1924

Abbot (Charles G.):

The Sun. (Revised Edition.)
(*Purchased.*) 8vo. New York, 1929

Alfani (G.):

Relazione delle Osservazioni solstiziali eseguite alla meridiana del Duomo di Firenze negli anni 1927 e 1928.
(*Author.*) 8vo. Firenze, 1929

Amateur Telescope Making: A collective work. Albert G. Ingalls, Editor. With a foreword by Dr. Harlow Shapley.
(*Purchased.*) 8vo. [New York], 1928

American Association of Variable Star Observers:
——: *Monthly Reports and Annual Report for the year 1929.
(*Association.*) 8vo. Northfields, 1929
——: Julian Day Calendar, 1930.
(*Association.*) 8vo. Oakland, Cal., [1930]

Arcetri, Osservatorio Astrofisico:
Immagini spettroscopiche del Bordo Solare osservate a Catania, Madrid, Zôsé e Zurigo negli anni 1925 e 1926. [Unione Astronomica Internazionale.]
(*Observatory.*) 4to Firenze, 1929

Arctowski (Henryk):
*Recherches sur les relations géothermiques de la région de Boryslaw.
(*Author.*) 4to. Madrid, 1929

Barcelona, Observatorio Fabra:
Boletín; Sección Astronómica, Tomo II., No. 1.
(*Observatory.*) 4to. Barcelona, 1930

Belgrade, Observatoire Astronomique de l'Université:
Publications, Tome 1, 2. (Annuaire 1929, 1930.)
(*Observatory.*) 4to. Beograd, 1928–29

Berlin, Preussische Akademie der Wissenschaften:
Geschichte des Fixsternhimmels. Abteilung I., Bände 11, 12.
(*Purchased.*) 4to. Karlsruhe, 1929–30
Band 11. 10^h Rektaszension.
Band 12. 11^h Rektaszension.

Bern, Schweizerische Geodätische Kommission:
——: Astronomisch-geodätische Arbeiten in der Schweiz, Band 18.
(*Swiss Geodetic Commission.*) 4to. Bern, 1929
——: Procès-Verbal de la 75^{me} Séance.
(*Swiss Geodetic Commission.*) 8vo. Neuchatel, 1929

Bern, Universität:
Schriften 1929 (1).
(*University.*) 8vo. Bern, 1929

Birmingham and Midland Institute Scientific Society:
Record 1872–1929 and Abstracts 1927–1929.
(*Institute.*) 8vo. Birmingham, 1929

Bologna, Reale Accademia delle Scienze dell' Istituto:
—— : Memorie, Sezione delle scienze fisiche e matematiche, Serie 8, Tomo 6.
(*Academy.*) 4to. Bologna, 1929
—— : Rendiconto della sessione 1928–29, Nuova serie, vol. 33.
(*Academy.*) 8vo. Bologna, 1929

Bordeaux, Observatoire:
Carte photographique du Ciel. Zones +12°, +14°, +16°. (15 Charts.)
(*Observatory.*)

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Tychonis Brahe Dani Opera Omnia: edidit J. L. E. Dreyer. Tomus XV. (Carminum Appendix; Index Hominum et Rerum ad tomos I.–XIV.)
(*Admiral Dreyer.*) 4to. Hauniae, 1929

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Constants for the Observing Stations.
(*Seismology Committee.*) 8vo. Ryde, I.O.W., 1929

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Memoirs, No. 1.
(*Director.*) 4to. Nairobi, 1929

British Science Guild:
A Catalogue of British Scientific and Technical Books covering every Branch of Science and Technology, carefully classified and indexed. New edition, entirely revised and enlarged.
(*Purchased.*) 8vo. London, 1925

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Astronomische Abhandlungen, Band 1, No. 1.
(*Observatory.*) 8vo. Budapest, 1929

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Jahrbücher, Bände 49, 50, 54–57 (1919–1927).
(*Institution.*) 4to. Budapest, 1927–29

Canada, Geodetic Survey:
Annual Report of the Superintendent . . . for the fiscal year ending March 31, 1928.
(*Superintendent.*) 8vo. Ottawa, 1929

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A Table of Coefficients to facilitate interpolation by means of the formulæ of Gauss, Bessel, and Everett. Computed by Edwin Chappell.
(*Author.*) 4to. London, 1929

Comrie (L. J.):
——: *Interpolation Tables.
(*Author.*) 8vo. London, 1929
——: *German Calculating Machine Enterprise.
(*Author.*) 16mo. [s.l.], 1929

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Classe des Sciences mathématiques et naturelles. Série A: Sciences mathématiques. Mémoires, Vol. 1.
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Bind 1, 2.
[Contains Tychoniana.]
(*Dr. W. Doberck.*) 8vo. Kjøbenhavn, 1745–46

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Schriften, Neue Folge, Band 18, Hefte 3, 4.
(*Society.*) 8vo. Dantzig, 1927–29

Darmstaedter (Ludwig):
Handbuch zur Geschichte der Naturwissenschaften und der Technik. In chronologischer Darstellung. Zweite, umgearbeitete und vermehrte Auflage. Unter Mitwirkung von Professor Dr. R. du Bois-Reymond und Oberst z. D. C. Schaefer, herausgegeben von Ludwig Darmstaedter.
(*Dr. E. B. Knobel.*) 8vo. Berlin, 1908

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Relazione sulla revisione della meridiana di S. M. del Fiore in Firenze e sulla istituzione di riferimenti per future verifiche di stabilità della cupola.
(*Prof. G. Alfani.*) 8vo. Firenze, 1929

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*The Variation of Latitude.
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Engineering: An Illustrated Weekly Journal:
Vol. 128, Nos. 3321, 3323, 3325, 3327.
Fol. London, 1929
(Containing a description of the 36-in. Reflector of the Royal Observatory, Edinburgh.)

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*Rapport de la Réunion de l'ozone et de l'absorption atmosphérique, avec 27 mémoires et notes mis en ordre par Charles Fabry, Paris.
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(*Purchased.*) 8vo. Roma, 1899

Fowler (R. H.):
Statistical Mechanics, the Theory of the Properties of Matter in Equilibrium. Based on an Essay awarded the Adams Prize in the University of Cambridge, 1923–24.
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Tyge Brahe. En historisk Fremstilling efter trykte og utrykte Kilder.
(*Dr. W. Doberck.*) 8vo. Kjøbenhavn, 1871

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*Determinazioni astronomiche delle Coordinate Geographiche di London (Spitzbergen) eseguite dai tenenti di Vascello Jurino Jannucci ed Ernesto Pellegrini.
(*Institute.*) fol. Genova, 1929

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(*Author.*) MS.
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——: *Plan der Errichtung einer deutschen Sternwarte in Windhuk.
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Band II., Erste Hälfte. Grundlagen der Astrophysik. Zweiter Teil. Bearbeitet von K. F. Bottlinger, A. Brill, E. Schoenberg, H. Rosenberg.
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Hartmann (Juan):
——: *Der Gregorianische Kalender und die Bestrebungen zu seiner Verbesserung.
(*Author.*) 4to. Roma, 1928
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*Variation of the Space-Velocity of Stars of Different Spectral Types with their Absolute Magnitudes.
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Hufnagel (Leon):
*Temperatures of Giants and Dwarfs.
(*Author.*) 4to. Cambridge (Mass.), 1929

International Astronomical Union:
——: Bulletin for Character Figures of Solar Phenomena. Published by the Eidgenossische Sternwarte in Zürich. Nos. 6–8.
(*I. A. U.*) 4to. Zürich, 1929
——: *Commission des télégrammes astronomiques, 1925–1928.
(*Prof. E. Strömgren.*) 4to. Cambridge, 1929
——: Délimitation scientifique des constellations (Tables et Cartes), par E. Delporte. Being the Report of Commission III.
(*Uccle Observatory.*) 4to. Cambridge, 1930

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(*International Research Council.*) 8vo. Paris, 1929

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*On the Temperature-Coefficients of the Micrometer-screws of the Large Zenith Telescopes used in the International Latitude Stations in the North Parallel.
(*Author.*) 4to. Tokyo, 1930

Kwiatkowski (A.) and J. Niewiarowski:
*Détermination des coordonnées géographiques de l'Observatoire de l'Institut d'Astronomie pratique de l'Ecole Polytechnique de Varsovie.
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Kyoto Imperial University, Kwasan Observatory:
Publications. Vol. 1, Nos. 1, 2.
(*Observatory.*) 8vo. Kyoto, 1929–30

Levi-Civita (Tullio):
A Simplified Presentation of Einstein's Unified Field Equations. Authorised Translation by John Dougall, M.A., D.Sc.
(*Purchased.*) 8vo. London and Glasgow, 1929

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A Syllabus of a Course of Lectures on Astronomy; illustrated by that magnificent and highly improved apparatus the new Dioastrodoxon; or grand transparent orrery, twenty-one feet diameter. . . . With an epitome of that branch of science, its origin, and progress, and a short account of that beautiful machine.
(*Mr. W. Van Sommer.*) 8vo. Oxford, 1816

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Publications. Vol. 5.
(*Laboratory.*) 8vo. Louvain, 1929

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Modern Cosmologies: a historical sketch of researches and theories concerning the structure of the universe.
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(*Prof. E. Neville.*) 8vo. London, 1929

Mauritius, Royal Alfred Observatory:
Miscellaneous Publications. Nos. 8–10.
(*Observatory.*) fol. [Port Louis, 1929–30]

Melbourne Observatory:
Astrographic Catalogue, 1900·0. Vol. 3. Zones −69°, −70°.
(*Observatory.*) 4to. Melbourne, [1929]

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——: Journal. Vol. 20, Nos. 1–3.
(*Purchased.*) 8vo. Menasha, 1930
——: Review of Scientific Instruments. New Series, Vol. 1, Nos. 1–3.
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(*Observatory.*) 8vo. Milano, 1929

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Milne (E. A.):
The Aims of Mathematical Physics. An inaugural lecture delivered before the University of Oxford on 19 November 1929.
(*Author.*) 8vo. Oxford, 1929

Milne-Thomson (L. M.):
Standard Table of Square Roots. The square roots to eight significant figures of all four-figure numbers, with printed differences.
(*Author.*) 8vo. London, 1929

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Astronomische Chronologie. Erste Band, Text: Zweiter Band, Tafeln.
(*Purchased.*) 2 vols. 8vo. Berlin u. Leipzig, 1929

Neumann (M. M. S.-Navarro):
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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XC. No. 1. NOVEMBER 1929.

PUBLISHED AND SOLD BY THE ROYAL ASTRONOMICAL SOCIETY,
BURLINGTON HOUSE, LONDON, W. 1.

PRICE TO NON-FELLOWS, 4*s.* 0*d.*

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Printed in Great Britain by NEILL & CO., LTD., Edinburgh, December 1929.